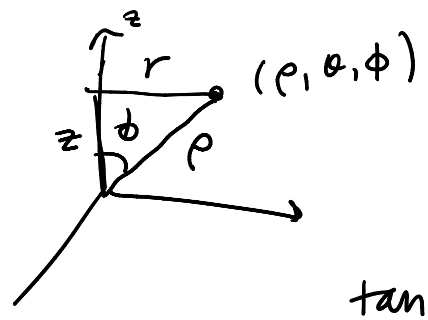
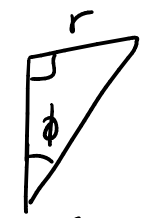
Review Exam 2 - 11/6/2023 - 4:30pm

Cone $\sqrt{3}\,z = \sqrt{x^2+y^2}$

$$z = \frac{r}{\sqrt{3}}$$





$$\tan\phi = \frac{r}{z} = \frac{r}{\left(\frac{r}{\sqrt{3}}\right)} = \sqrt{3}$$

$$\tan\phi = \sqrt{3}$$

$$\phi = \frac{\pi}{3}$$

$$x - y = 0$$
$$y = x$$

$$x = \cancel{\rho\sin\phi}\cos\theta = y = \cancel{\rho\sin\phi}\sin\theta$$
$$1 = \tan\theta$$
$$\theta = \frac{\pi}{4}$$

☆ Lagrange Multipliers

min/max $f(x,y,z)$

subj to $g(x,y,z) = 0$

4 equations
4 unknowns
$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = 0 \end{cases} \Rightarrow$$

Solve for (x,y,z)
plug (x,y,z) into f to look for max/min

$x = +y$ in $g = 2x^2 + 3xy + 2y^2 = 7$

$y = +x$

$$2x^2 + 3x(x) + 2(x)^2 = 7x^2 = 7$$
$$x = \pm 1 = y$$
$$(1,1) \quad (-1,-1)$$

$y = -x$

$$2x^2 + 3x(-x) + 2(-x)^2 = 7$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$

$\rightarrow$)

$x^2 = 7$

$x = \pm\sqrt{7}$

$(\sqrt{7}, -\sqrt{7}), (-\sqrt{7}, \sqrt{7})$

Cartesian vs. Polar

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$$

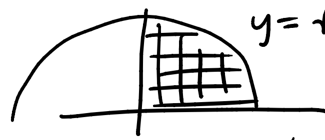
$$= \int_0^1 \left[\frac{y^3}{3} \right]_0^{\sqrt{1-x^2}} dx$$

$$= \frac{1}{3} \int_0^1 (1-x^2)^{3/2} \, dx$$

Trig Sub $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$$= \frac{1}{3} \int_0^{\frac{\pi}{2}} \cos^3\theta \cos\theta \, d\theta$$



$$= \int_0^{\frac{\pi}{2}} \int_0^1 r^2 \sin^2\theta \; r \, dr \, d\theta$$

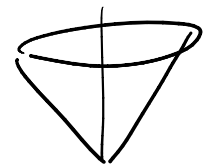
use geometry to replace trig sub.

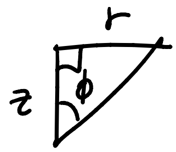
Q: Which coordinate system to use?

	Cartesian	Cylindrical	Spherical
Domain	Planes $ax + by + cz = d$ Tetrahedron Box	$r = b$ $z = cr$ $z^2 + r^2 = b^2$ elliptic paraboloid $z = ar^2$	$\rho = a$ $\phi = \beta$ $z = 0$ $\phi = \frac{\pi}{2}$ $\theta = \alpha$
Bounds $\int_a^b$	$z = d - ax - by$ $z = \sin(x)$	$z = \sqrt{x^2+y^2}$ $y = \sqrt{1-x^2}$ look like a trig sub	$z = \sqrt{1-x^2-y^2}$
Integrand	all others	$f(x^2+y^2)$ polynomial x, xy	$f(x^2+y^2+z^2)$ polynomial

Integrand | all others | polynomial x, xy | polynomial

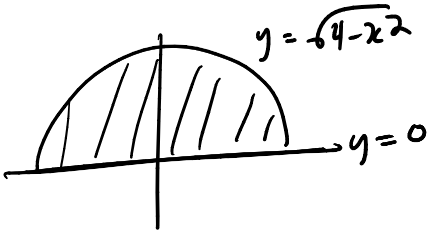
$$z = \sqrt{x^2+y^2} = r$$



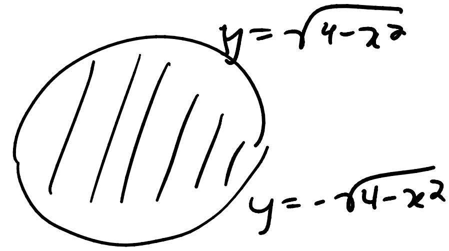


$$\tan\phi = \frac{r}{z} = \frac{r}{r} = 1$$

$$\phi = \frac{\pi}{4}$$



$0 \le \theta \le \pi$



$0 \le \theta \le 2\pi$

★ **Vector Fields:** $\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$

conservative if $\vec{F} = \nabla\phi$

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

$$\phi = \int P(x,y)\,dx = \int Q(x,y)\,dy$$

$\vec{F} = \langle P, Q, R \rangle$ is conservative if

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \qquad \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z} \qquad \frac{\partial R}{\partial y} = \frac{\partial Q}{\partial z}$$

$$\langle u, v \rangle = \left\langle \frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y} \right\rangle$$

$$F = \langle y, x \rangle = \left\langle \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right\rangle$$

$$\phi = \int y \, dx = xy + \cancel{a(y)}$$

$$= \int x \, dy = xy + \cancel{b(x)} \quad = \quad xy + C$$

$$\frac{\partial}{\partial x}(-y) = 0$$