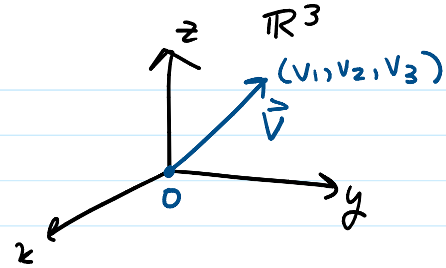
Review of Vectors



$\mathbb{R}^3$ - denotes 3D space

position vector: $\vec{v} = \langle v_1, v_2, v_3 \rangle$

magnitude of $\vec{v}$:

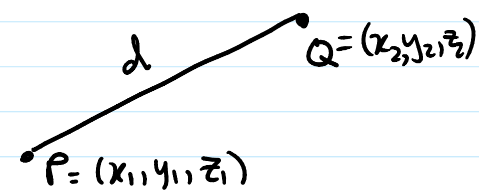
$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

unit vector: a vector $\vec{u}$ with magnitude $|\vec{u}| = 1$

coordinate unit vectors:

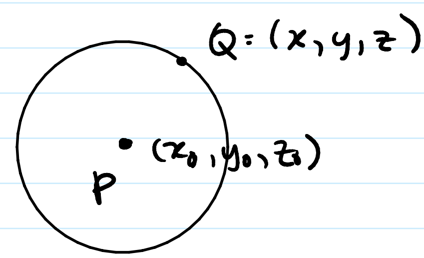
$$\hat{i} = \langle 1, 0, 0 \rangle \qquad \hat{j} = \langle 0, 1, 0 \rangle \qquad \hat{k} = \langle 0, 0, 1 \rangle$$

distance between two points P and Q



$$d = |\vec{PQ}|$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



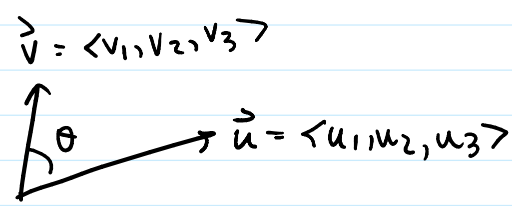
equation of sphere:

radius r

center (x_0, y_0, z_0)

sphere is the set of all points Q such that $|\vec{PQ}|^2 = r^2$

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$



dot product of $\vec{u}$ and $\vec{v}$:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$= |\vec{u}|\,|\vec{v}| \cos\theta$$

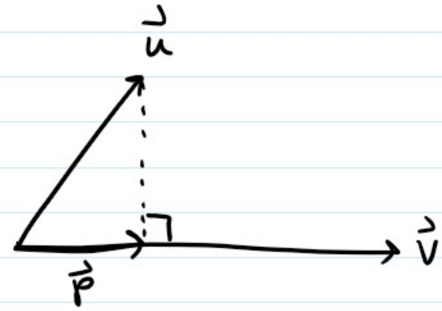
the dot product is a scalar

orthogonal: $\vec{u}$ and $\vec{v}$ are orthogonal if and only if

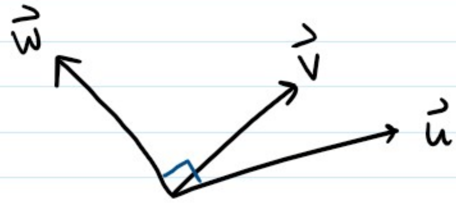
$$\vec{u} \cdot \vec{v} = 0$$

orthogonal projection of $\vec{u}$ onto $\vec{v}$:

$$\vec{p} = \text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right)\vec{v}$$



cross product of $\vec{u}$ and $\vec{v}$:



$$\vec{w} = \vec{u} \times \vec{v}$$

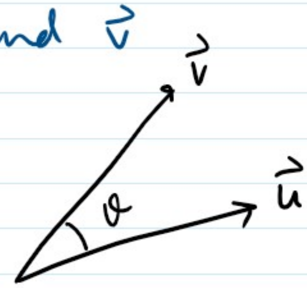
$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \hat{\imath}\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + \hat{k}\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$= (u_2v_3 - u_3v_2)\hat{\imath} - (u_1v_3 - u_3v_1)\hat{\jmath} + (u_1v_2 - v_1u_2)\hat{k}$$

$\vec{w}$ is orthogonal to both $\vec{u}$ and $\vec{v}$

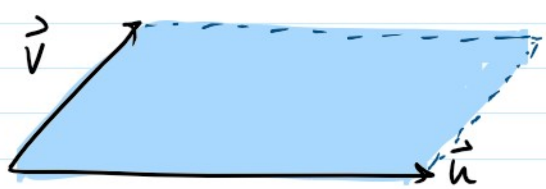
magnitude of the cross product:

$$|\vec{u} \times \vec{v}| = |\vec{u}|\,|\vec{v}| \sin\theta$$



Area of parallelogram with sides $\vec{u}$ and $\vec{v}$:

$$A = |\vec{u} \times \vec{v}|$$



Area of triangle with sides $\vec{u}$ and $\vec{v}$:

$$A = \frac{1}{2} \left| \vec{u} \times \vec{v} \right|$$