Quiz Grading Rubric: MA 261 Fall 2023

September 1, 2023

Rubric

Category	Score	Description
No Response	0	Either the work is (a) not attempted or (b) incorrect, irrelevant, or off task.
Minimal	1	Work demonstrates only a minimal understanding and a reasonable approach is not suggested. Response is incomplete, contains major mathematical errors, or reveals serious flaws in reasoning. Parts may be absent or irrelevant.
Initiated	2	Work contains evidence of a conceptual understanding and a reasonable approach is indicated. However, work is unable to follow through or complete the appropriate mathematics. Parts may be absent or irrelevant.
Partial	3	Work contains evidence of a conceptual understanding and a reasonable approach is indicated. However, on the whole, the response is not well developed. Response contains some correct mathematics but there are serious mathematical errors or flaws in reasoning. Parts are all attempted.
Satisfactory	4	The response demonstrates a clear understanding and provides an acceptable approach. However, it contains omissions or minor errors in mathematics. Parts are attempted but may not be completely developed.
Excellent	5	The response demonstrates a complete understanding of the problem, is correct, and the methods of solution are appropriate and fully developed. The response is logically sound, clearly written, and does not contain any significant errors. Parts are all completed.

Note

For the 5-point question, the student must show their work to receive full credit. If a student merely writes down the letter corresponding to an answer, they will receive only 1 point, even if that answer is correct. Students must demonstrate that they know how to arrive at the correct answer in order to receive full credit.

Example: No Response (0 points)

Either the work is (a) not attempted or (b) incorrect, irrelevant, or off task.

Question: Find the equation of the plane through the point $(0, 1, 2)$ perpendicular to the planes given by $x - y + 2z = 1$ and $3x + 2z = -4$.

Student Answer:

(a) Paper is blank, or

(b) Restates the problem:

$$x - y + 2z = 1 \quad \perp \quad 3x + 2z = -4$$

(c) Or work is irrelevant:

$$\vec{n} = \text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right) \vec{v} = \left(\frac{\langle 1, -1, 2\rangle \cdot \langle 3, 0, 2\rangle}{\langle 3, 0, 2\rangle \cdot \langle 3, 0, 2\rangle}\right) \langle 3, 0, 2\rangle = \left(\frac{7}{13}\right) \langle 3, 0, 2\rangle$$

Example: Minimal (1 point)

Work demonstrates only a minimal understanding and a reasonable approach is not suggested. Response is incomplete, contains major mathematical errors, or reveals serious flaws in reasoning. Parts may be absent or irrelevant.

Question: Find the equation of the plane through the point $(0, 1, 2)$ perpendicular to the planes given by $x - y + 2z = 1$ and $3x + 2z = -4$.

Student Answer:

normal vector:

$$\vec{n} \cdot \langle 1, -1, 2\rangle = 0$$
$$\vec{n} \cdot \langle 3, 0, 2\rangle = 0$$

Student recognizes that they need to find a normal vector but they use the dot product instead of the cross product. Student is unable to find the normal vector and does not set up the equation of the plane.

Student Answer: D

For the 5-point question, the student must show their work to receive full credit. If a student merely writes down the letter corresponding to an answer, they will receive only 1 point, even if that answer is correct. Students must demonstrate that they know how to arrive at the correct answer in order to receive full credit.

Example: Initiated (2 points)

Work contains evidence of a conceptual understanding and a reasonable approach is indicated. However, work is unable to follow through or complete the appropriate mathematics. Parts may be absent or irrelevant.

Question: Find the equation of the plane through the point $(0, 1, 2)$ perpendicular to the planes given by $x - y + 2z = 1$ and $3x + 2z = -4$.

Student Answer:

Student recognizes that they need a normal vector and knows the general equation of a plane.

normal vector:

$$\vec{n} = \langle 1, -1, 2\rangle \times \langle 3, 0, 2\rangle = \langle a, b, c\rangle$$

Student doesn't even attempt to find the cross product, or the attempt is nonsensical.

equation of the plane is $\vec{n} \cdot (\vec{x} - \vec{x_0}) = 0$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Student does not recognize that $\vec{x_0} = \langle 0, 1, 2 \rangle$

So the final answer is:

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

Student writes the correct form of the solution, but did not calculate the normal vector or recognize the value of $\vec{x_0}$.

Example: Partial (3 points)

Work contains evidence of a conceptual understanding and a reasonable approach is indicated. However, on the whole, the response is not well developed. Response contains some correct mathematics but there are serious mathematical errors or flaws in reasoning. Parts are all attempted.

Question: Find the equation of the plane through the point $(0, 1, 2)$ perpendicular to the planes given by $x - y + 2z = 1$ and $3x + 2z = -4$.

Student Answer:

Student recognizes that they need to use the cross product but doesn't remember how to calculate it.

normal vector:

$\vec{n} = \langle 1, -1, 2 \rangle \times \langle 3, 0, 2 \rangle$ =??? I forget how to do this

student tries to use dot product instead but fails to find the normal vector

guess that the normal vector is: $\vec{n} = \langle 1, 1, 0 \rangle$

equation of the plane through $(0, 1, 2)$ is $\vec{n} \cdot (\vec{x} - \vec{x_0}) = 0$

$$\begin{aligned} 1(x - 0) + 1(y - 1) + 0(z - 2) &= 0 \\ x + y - 1 + 0 &= 0 \\ x + y &= 1 \end{aligned}$$

So the final solution is:

$$\boxed{x + y = 1}$$

Student has a conceptual understanding and takes the right approach. Half of the solution is correct. The other half has the right approach but student did not calculate correctly.

Example: Acceptable (4 points)

The response demonstrates a clear understanding and provides an acceptable approach. However, it contains omissions or minor errors in mathematics. Parts are attempted but may not be completely developed.

Question: Find the equation of the plane through the point $(0, 1, 2)$ perpendicular to the planes given by $x - y + 2z = 1$ and $3x + 2z = -4$.

Student Answer:

Logic is correct, however the student makes a mathematical error that propagates so that the final solution is incorrect.

normal vector:

$$\begin{aligned}\vec{n} = \langle 1,-1,2\rangle \times \langle 3,0,2\rangle &= \det\begin{bmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & -1 & 2\\ 3 & 0 & 2\end{bmatrix}\\ &= \hat{\mathbf{i}}\det\begin{bmatrix}-1 & 2\\ 1 & 2\end{bmatrix} - \hat{\mathbf{j}}\det\begin{bmatrix}-1 & 2\\ 3 & 2\end{bmatrix} + \hat{\mathbf{k}}\det\begin{bmatrix}1 & -1\\ 3 & 0\end{bmatrix}\\ &= \hat{\mathbf{i}}(-2-0) - \hat{\mathbf{j}}(2-6) + \hat{\mathbf{k}}(0+3)\\ &= \langle -2,4,3\rangle\end{aligned}$$

Student calculates the normal vector correctly.

equation of the plane through $(0,1,2)$ is $\vec{n}\cdot(\vec{x}-\vec{x_0}) = 0$

$$\begin{aligned}0(x+2)+1(y-4)+2(z-3) &= 0\\ y-4+2z-6 &= 0\\ y+2z &= 4+6 = 10\end{aligned}$$

Student mixes up the normal vector $\vec{n}$ and the point $\vec{x_0}$

So the final answer is:

$$\boxed{y+2z=10}$$

The final answer is incorrect, but the approach is mostly correct with a small conceptual error and the student's answer has similar form to the true solution.

Example: Excellent (5 points)

The response demonstrates a complete understanding of the problem, is correct, and the methods of solution are appropriate and fully developed. The response is logically sound, clearly written, and does not contain any significant errors. Parts are all completed.

Question: Find the equation of the plane through the point $(0,1,2)$ perpendicular to the planes given by $x-y+2z=1$ and $3x+2z=-4$.

Student Answer:

normal vector:

$$\begin{aligned}\vec{n} = \langle 1,-1,2\rangle \times \langle 3,0,2\rangle &= \det\begin{bmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & -1 & 2\\ 3 & 0 & 2\end{bmatrix}\\ &= \hat{\mathbf{i}}\det\begin{bmatrix}-1 & 2\\ 1 & 2\end{bmatrix} - \hat{\mathbf{j}}\det\begin{bmatrix}-1 & 2\\ 3 & 2\end{bmatrix} + \hat{\mathbf{k}}\det\begin{bmatrix}1 & -1\\ 3 & 0\end{bmatrix}\\ &= \hat{\mathbf{i}}(-2-0) - \hat{\mathbf{j}}(2-6) + \hat{\mathbf{k}}(0+3)\\ &= \langle -2,4,3\rangle\end{aligned}$$

equation of the plane through $(0,1,2)$ is $\vec{n}\cdot(\vec{x}-\vec{x_0}) = 0$

$$\begin{aligned}-2(x-0)+4(y-1)+3(z-2) &= 0\\ -2x+4y-4+3z-6 &= 0\\ -2x+4y+3z &= 4+6 = 10\end{aligned}$$

So the final answer is:

$$\boxed{-2x + 4y + 3z = 10}$$

The answer is correct. Students shows all their steps and their logic is sound. May be off by a minus sign or small factor.