

Example

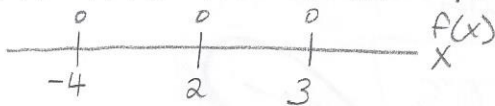
Find a polynomial of degree 4 with

i) Zeros at $x = -4, x = 3, x = 2$

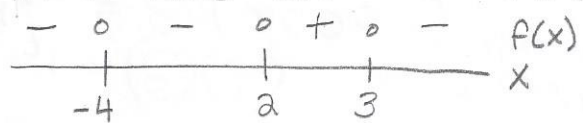
ii) $f(0) = -16$

iii) $f(x) > 0$ only on $(2, 3)$

• Start with the number line with the zeros labelled:



• From (iii), we can label the intervals where $f(x)$ is positive and negative:



• Since the sign on $f(x)$ does not change at the zero $x = -4$, this zero has even multiplicity. Since the sign on $f(x)$ does change at the zeros $x = 2$ and $x = 3$, these zeros have odd multiplicity.

• Then $f(x) = k(x+4)^{\overset{\text{even}}{2}}(x-2)^{\overset{\text{odd}}{1}}(x-3)^{\overset{\text{odd}}{1}}$.
Note $f(x)$ has degree 4.

• Now use (ii) to find k .

$$f(0) = -16$$

$$f(0) = k(0+4)^2(0-2)(0-3)$$

$$-16 = k(16)(-2)(-3)$$

$$-16 = k(16)(6)$$

$$-1 = 6k$$

$$-\frac{1}{6} = k$$

• Thus, $f(x) = -\frac{1}{6}(x+4)^2(x-2)(x-3)$