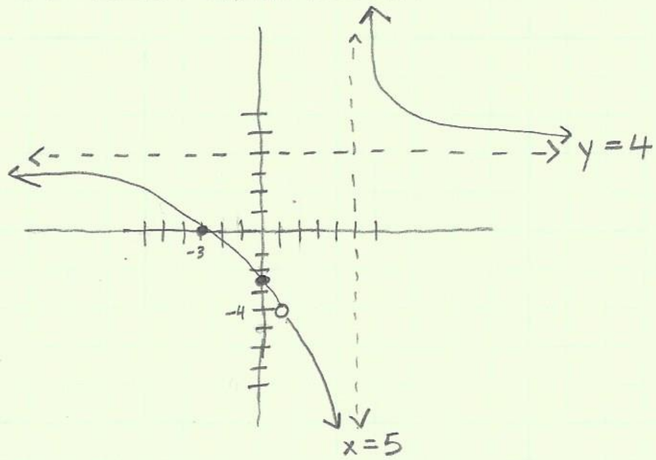
Ex.1 (a) Sketch a graph for

$$f(x) = \frac{4x^2+8x-12}{x^2-6x+5}$$
$$= \frac{4(x^2+2x-3)}{(x-5)(x-1)}$$
$$= \frac{4(x+3)\cancel{(x-1)}}{(x-5)\cancel{(x-1)}}$$
$$= \frac{4(x+3)}{x-5}$$

Zeros: $x=-3$
y-int: $f(0) = -\frac{12}{5}$
Hole: at $x=1$, $y = \frac{4(4)}{-4} = -4$
VA: $x=5$
HA: $y=4$
SA: DNE
Point where f(x) crosses HA:
$$4 = \frac{4(x+3)}{x-5}$$
$$4x-20 = 4x+12$$
$$\cancel{-20 = 12}$$
DNE

Plot found information:



Note: • The function will "hug" the asymptotes.
• We only cross the x-axis at the zeros.
• Never cross VA.
• Plug in x-values as necessary to get a better idea of f(x).

(b) Use the graph to find the following:
(Exclude holes and VA, only exclude HA if not crossed)

Domain: Travel from left to right and exclude any x-values where the function is not defined.
• $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$

Range: Travel from bottom to top and exclude any y-values where the function is not defined.
• $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

Intervals where f(x) is decreasing: ($\searrow$ - an arrow should be travelling downward along the function, or rather, as the x-values increase, the (f(x) or) y-values decrease)
• $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$

Intervals where f(x) is increasing: ($\nearrow$ - an arrow should be travelling upward along the function, or rather, as the x-values increase, the (f(x) or) y-values increase)
• DNE

Intervals where $f(x) > 0$ (graph is above x-axis): $(-\infty, -3) \cup (5, \infty)$

Intervals where $f(x) < 0$ (graph is below x-axis): $(-3, 1) \cup (1, 5)$

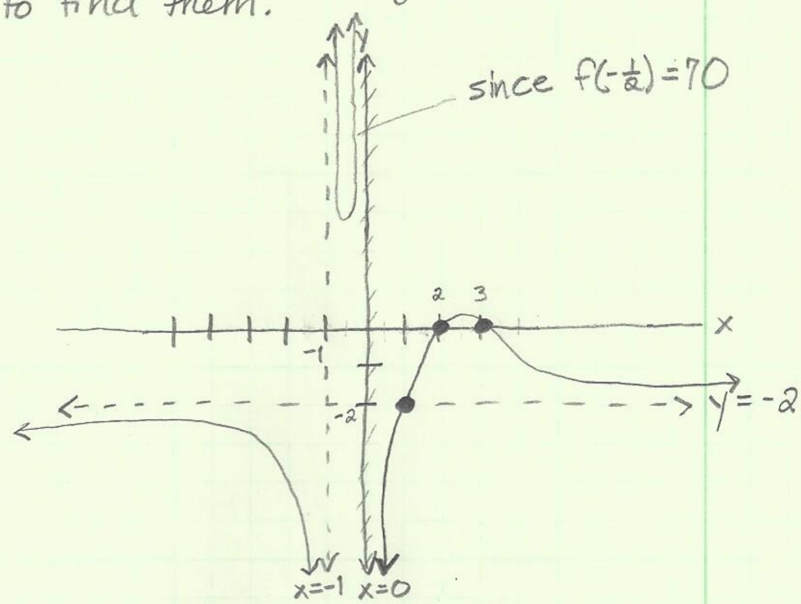
Note: For some functions, we can't find the range or increasing/decreasing intervals without using calculus, so you won't be asked to find them.

Ex. 2 (a) Sketch a graph of

$$f(x) = \frac{-2x^2+10x-12}{x^2+x}$$

$$= \frac{-2(x^2-5x+6)}{x(x+1)}$$

$$= \frac{-2(x-3)(x-2)}{x(x+1)}$$



Zeros: $x=2$, $x=3$

y-int: DNE (division by 0)

Holes: DNE

VA: $x=0$, $x=-1$

HA: $y=-2$

SA: DNE

Point at which $f(x)$ crosses HA:

$$-2 = \frac{-2x^2+10x-12}{x^2+x}$$

$$-2x^2-2x = -2x^2+10x-12$$

$$-12x = -12$$

$$x = 1 \qquad (1,-2)$$

(b) Domain: $(-\infty,-1) \cup (-1,0) \cup (0,\infty)$

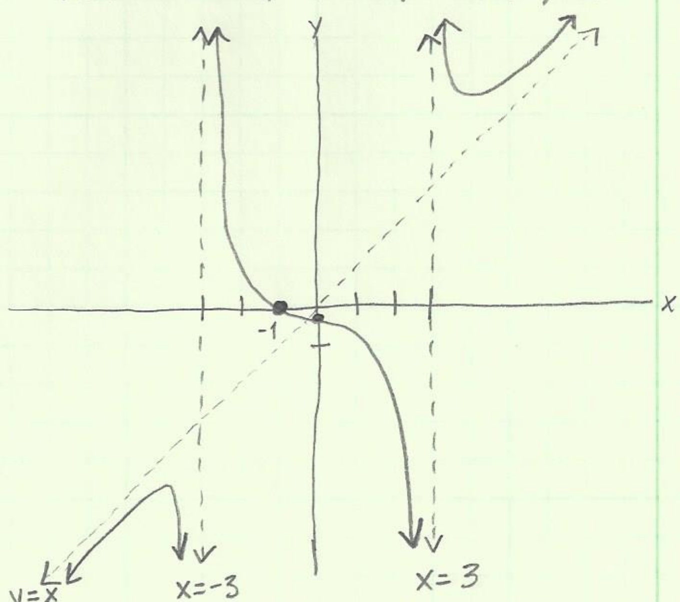
$f(x)>0$: $(-1,0) \cup (2,3)$

$f(x)<0$: $(-\infty,-1) \cup (0,2) \cup (3,\infty)$

Ex. 3 (a) Sketch a graph of

$$f(x) = \frac{x^3+1}{x^2-9}$$

$$= \frac{(x+1)(x^2-x+1)}{(x-3)(x+3)}$$



Can find middle of graph by calculating $f(-2)$ and $f(2)$

Zeros: $x=-1$, ~~$x=\frac{1\pm\sqrt{1-4}}{2}$~~

y-int: $f(0) = -\frac{1}{9}$

Holes: DNE

VA: $x=-3$, $x=3$

HA: DNE

SA:

$$\begin{array}{r} x \\ x^2-9 \overline{) x^3+0x^2+0x+1} \\ -(x^3 -9x) \downarrow \\ \hline 9x+1 \end{array}$$

$$f(x) = x + \frac{9x+1}{x^2-9}$$

$\boxed{y=x}$

Point where $f(x)$ crosses HA: DNE

(b) Domain: $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$

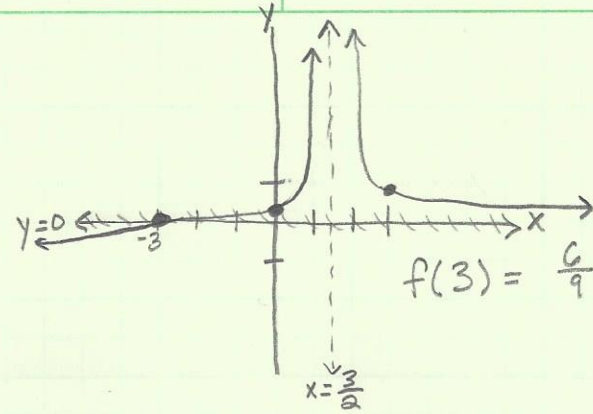
Range: $(-\infty,\infty)$

$f(x)>0$: $(-3,-1) \cup (3,\infty)$

$f(x)<0$: $(-\infty,-3) \cup (-1,3)$

Ex. 4 (a) Sketch a graph of

$$f(x) = \frac{x+3}{(2x-3)^2}$$



$f(3) = \frac{6}{9} = \frac{2}{3}$

Zeros: $x = -3$

y-int: $f(0) = \frac{1}{3}$

Holes: DNE

VA: $x = \frac{3}{2}$

HA: $y = 0$

SA: DNE

Point at which $f(x)$ crosses HA:

$0 = \frac{x+3}{(2x-3)^2}$

$0 = x+3$

$x = -3$

$(-3, 0)$

(b) Domain: $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

Increasing: $(-\infty, \frac{3}{2})$

Decreasing: $(\frac{3}{2}, \infty)$

$f(x) > 0$: $(-3, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

$f(x) < 0$: $(-\infty, -3)$

Ex. 5 (a) Sketch a graph of

$$f(x) = \frac{4x^3+4x^2-4x-4}{x^2-x-2}$$

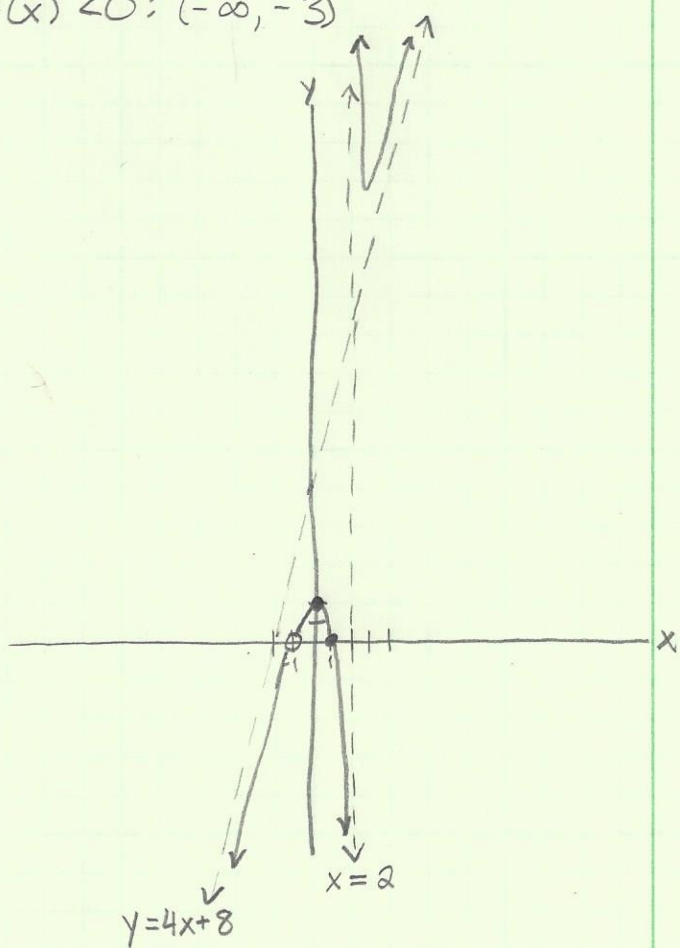
$$= \frac{4(x^3+x^2-x-1)}{(x-2)(x+1)}$$

$$= \frac{4(x^2(x+1)-1(x+1))}{(x-2)(x+1)}$$

$$= \frac{4(x^2-1)(x+1)}{(x-2)(x+1)}$$

$$= \frac{4(x+1)(x-1)\cancel{(x+1)}}{(x-2)\cancel{(x+1)}}$$

$$= \frac{4(x+1)(x-1)}{x-2}$$



hole!

Zeros: $x = 1$, $\cancel{x = -1}$

y-int: $f(0) = 2$

Holes: @ $x = -1$

$y = \frac{4(-1+1)(-1-1)}{-1-2} = 0$

$(-1, 0)$

VA: $x = 2$

HA: DNE

SA:

$$\begin{array}{r|l} & 4x + 8 \\ x-2 & 4x^2 + 0x - 4 \\ & -(4x^2 - 8x) \downarrow \\ & \quad 8x - 4 \\ & \quad -(8x - 16) \\ & \quad\quad 12 \end{array}$$

$f(x) = 4x + 8 + \frac{12}{x-2}$

$\boxed{y = 4x+8}$

Point where $f(x)$ crosses HA: DNE

(b) Domain: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

$f(x) > 0$: $(-1, 1) \cup (2, \infty)$

$f(x) < 0$: $(-\infty, -1) \cup (1, 2)$