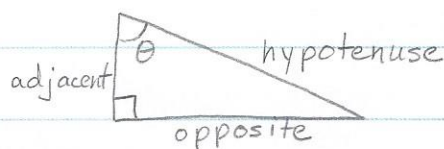
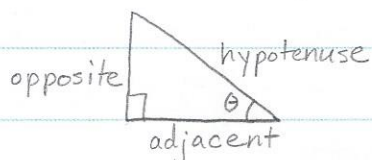


Lesson 21: Trigonometric Functions



Sine: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

Cosine: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$

Tangent: $\tan \theta = \frac{\text{opp}}{\text{adj}}$

Cosecant: $\csc \theta = \frac{\text{hyp}}{\text{opp}}$

Secant: $\sec \theta = \frac{\text{hyp}}{\text{adj}}$

Cotangent: $\cot \theta = \frac{\text{adj}}{\text{opp}}$

SOH
h p h

CAH
s d p

TOA
a p d

Note:

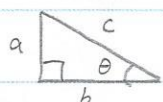
$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

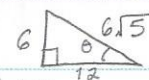
$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Ex. 1



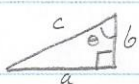
, $a = 6$, $b = 12$, find all 6 trig functions for θ .

$$a^2 + b^2 = c^2 \Rightarrow 36 + 144 = 180 = c^2 \Rightarrow c = \sqrt{180} = 6\sqrt{5}$$



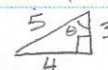
$$\sin \theta = \frac{6}{6\sqrt{5}} \quad \cos \theta = \frac{12}{6\sqrt{5}} \quad \tan \theta = \frac{6}{12} \quad \csc \theta = \frac{6\sqrt{5}}{6} \quad \sec \theta = \frac{6\sqrt{5}}{12} \quad \cot \theta = \frac{12}{6}$$

Ex. 2



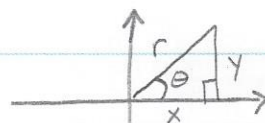
, $b = 3$, $c = 5$, find all 6 trig functions for θ .

$$a^2 + b^2 = c^2 \Rightarrow a^2 = c^2 - b^2 = 25 - 9 = 16 \Rightarrow a = 4$$



$$\sin \theta = \frac{4}{5} \quad \cos \theta = \frac{3}{5} \quad \tan \theta = \frac{4}{3} \quad \csc \theta = \frac{5}{4} \quad \sec \theta = \frac{5}{3} \quad \cot \theta = \frac{3}{4}$$

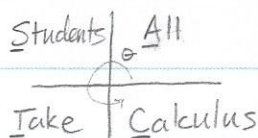
* In the coordinate plane:



$$r^2 = x^2 + y^2$$

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x} \quad \csc \theta = \frac{r}{y} \quad \sec \theta = \frac{r}{x} \quad \cot \theta = \frac{x}{y}$$

Signs:



QII (-, +)
 $\sin \theta > 0$
 $\cos \theta < 0$
 $\tan \theta < 0$

QI (+, +)
 $\sin \theta > 0$
 $\cos \theta > 0$
 $\tan \theta > 0$

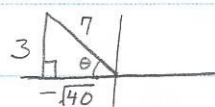
$$\Rightarrow \begin{aligned} \csc \theta &> 0 \\ \sec \theta &> 0 \\ \cot \theta &> 0 \end{aligned}$$

QIII (-, -)
 $\sin \theta < 0$
 $\cos \theta < 0$
 $\tan \theta > 0$

QIV (+, -)
 $\sin \theta < 0$
 $\cos \theta > 0$
 $\tan \theta < 0$

$$= \frac{\text{opp}}{\text{hyp}}$$

Ex.3 Given $\sin \theta = \frac{3}{7}$ and $\cos \theta < 0$, find the remaining 5 trig functions.

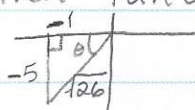


$$x = -\sqrt{49-9} = -\sqrt{40}$$

$$\begin{aligned} \sin \theta &= \frac{3}{7} & \cos \theta &= -\frac{\sqrt{40}}{7} & \tan \theta &= \frac{3}{-\sqrt{40}} \\ \csc \theta &= \frac{7}{3} & \sec \theta &= -\frac{7}{\sqrt{40}} & \cot \theta &= \frac{-\sqrt{40}}{3} \end{aligned}$$

$$= \frac{\text{opp}}{\text{adj}}$$

Ex.4 Given $\tan \theta = 5 = \frac{5}{1}$ and $\sec \theta < 0$, find the remaining 5 trig functions.

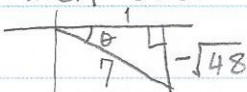


$$r^2 = 5^2 + 1^2 = 26 \Rightarrow r = \sqrt{26}$$

$$\begin{aligned} \sin \theta &= \frac{-5}{\sqrt{26}} & \cos \theta &= \frac{-1}{\sqrt{26}} & \tan \theta &= \frac{-5}{-1} = 5 \\ \csc \theta &= \frac{\sqrt{26}}{-5} & \sec \theta &= -\sqrt{26} & \cot \theta &= \frac{-1}{-5} = \frac{1}{5} \end{aligned}$$

$$= \frac{\text{hyp}}{\text{adj}}$$

Ex.5 Given $\sec \theta = 7$ in Quadrant IV, find the remaining 5 trig functions.

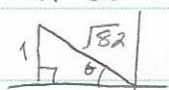


$$y = -\sqrt{49-1} = -\sqrt{48}$$

$$\begin{aligned} \sin \theta &= -\frac{\sqrt{48}}{7} & \cos \theta &= \frac{1}{7} & \tan \theta &= -\frac{\sqrt{48}}{1} \\ \csc \theta &= -\frac{7}{\sqrt{48}} & \sec \theta &= \frac{7}{1} = 7 & \cot \theta &= \frac{1}{-\sqrt{48}} \end{aligned}$$

$$= \frac{\text{adj}}{\text{opp}}$$

Ex.6 Given $\cot \theta = -9$ and $\sin \theta > 0$, find the remaining 5 trigs.

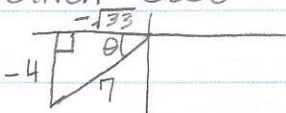


$$r = \sqrt{81+1} = \sqrt{82}$$

$$\begin{aligned} \sin \theta &= \frac{1}{\sqrt{82}} & \cos \theta &= -\frac{9}{\sqrt{82}} & \tan \theta &= -\frac{1}{9} \\ \csc \theta &= \sqrt{82} & \sec \theta &= -\frac{\sqrt{82}}{9} & \cot \theta &= -9 \end{aligned}$$

$$= \frac{\text{hyp}}{\text{opp}}$$

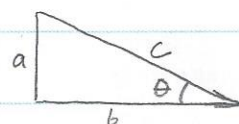
Ex.7 Given $\csc \theta = -\frac{7}{4}$ in QIII, find the remaining 5 trig functions.



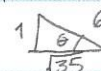
$$x = -\sqrt{49-16} = -\sqrt{33}$$

$$\begin{aligned} \sin \theta &= -\frac{4}{7} & \cos \theta &= \frac{-\sqrt{33}}{7} & \tan \theta &= \frac{-4}{-\sqrt{33}} \\ \csc \theta &= -\frac{7}{4} & \sec \theta &= \frac{7}{-\sqrt{33}} & \cot \theta &= \frac{-\sqrt{33}}{-4} \end{aligned}$$

Ex.8 Given $a = 1$ and $c = 6$, find all 6 trig functions for θ .



$$b = \sqrt{36-1} = \sqrt{35}$$



$$\begin{aligned} \sin \theta &= \frac{1}{6} & \cos \theta &= \frac{\sqrt{35}}{6} & \tan \theta &= \frac{1}{\sqrt{35}} \\ \csc \theta &= 6 & \sec \theta &= \frac{6}{\sqrt{35}} & \cot \theta &= \sqrt{35} \end{aligned}$$