

Lesson 24: Inverse Trig Functions

Inverse Trig Function	Domain	Range
$\arccos x = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\arcsin x = \sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\arctan x = \tan^{-1} x$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\operatorname{arccot} x = \cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$
$\operatorname{arcsec} x = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
$\operatorname{arccsc} x = \csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$

Note: Input x for $\sin x$ is an angle.
Output $\sin^{-1} x$ is an angle.

Ex.1 Approximate degree measure of acute θ to nearest minute. (θ_R reference angle)

(a) $\sin \theta = 0.3466$

$\Rightarrow \theta = \sin^{-1}(0.3466)$

In Calculator (DEG mode):

0.3466 $\boxed{2nd}$ $\boxed{SIN}$ $\boxed{2nd}$ $\boxed{=}$
↑ ↑
angle in DD angle in DMS

$\Rightarrow \theta = 20^\circ 17'$

(b) $\cos \theta = 0.4536 \Rightarrow \theta = \cos^{-1}(0.4536) = 63^\circ 2'$

(c) $\tan \theta = 2.902 \Rightarrow \theta = \tan^{-1}(2.902) = 70^\circ 59'$

(d) $\cot \theta = 4.497 \Rightarrow \frac{1}{\tan \theta} = 4.497 \Rightarrow \tan \theta = \frac{1}{4.497}$
 $\Rightarrow \theta = \tan^{-1}(\frac{1}{4.497})$
reciprocal angle in DD angle in DMS

In Calc (DEG mode): 4.497 $\boxed{1/x}$ $\boxed{2nd}$ $\boxed{TAN}$ $\boxed{2nd}$ $\boxed{=}$

$\theta = 12^\circ 32'$

$$(e) \sec \theta = 4.6955 \Rightarrow \frac{1}{\cos \theta} = 4.6955 \Rightarrow \cos \theta = \frac{1}{4.6955} \\ \Rightarrow \theta = \cos^{-1} \left(\frac{1}{4.6955} \right) = 77^\circ 42'$$

$$(f) \csc \theta = 4.6556 \Rightarrow \frac{1}{\sin \theta} = 4.6556 \Rightarrow \sin \theta = \frac{1}{4.6556} \\ \Rightarrow \theta = \sin^{-1} \left(\frac{1}{4.6556} \right) = 12^\circ 24'$$

Ex.2 Approximate radian measure of acute angle θ to 2 decimal places. (θ_R)

$$(a) \sin \theta = 0.5137 \Rightarrow \theta = \sin^{-1}(0.5137) = 0.54$$

$$(b) \cos \theta = 0.0468 \Rightarrow \theta = \cos^{-1}(0.0468) = 1.52$$

$$(c) \tan \theta = 2.0834 \Rightarrow \theta = \tan^{-1}(2.0834) = 1.12$$

$$(d) \cot \theta = 1.1241 \Rightarrow \theta = \tan^{-1} \left(\frac{1}{1.1241} \right) = 0.73$$

$$(e) \sec \theta = 3.0146 \Rightarrow \theta = \cos^{-1} \left(\frac{1}{3.0146} \right) = 1.23$$

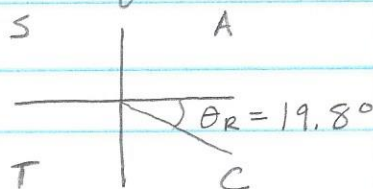
$$(f) \csc \theta = 3.7778 \Rightarrow \theta = \sin^{-1} \left(\frac{1}{3.7778} \right) = 0.27$$

Ex.3 Approximate to nearest 0.1° all angles in $[0^\circ, 360^\circ)$ that satisfy the equations.

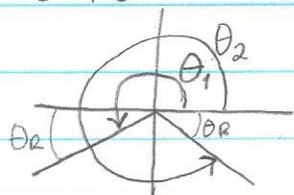
$$(a) \sin \theta = -0.3394$$

$$\theta = \sin^{-1}(-0.3394)$$

$$\theta = -19.8^\circ$$



$\sin \theta$ is negative in QIII and QIV, so



$$\theta_1 = 180^\circ + \theta_R = 180^\circ + 19.8^\circ = 199.8^\circ$$

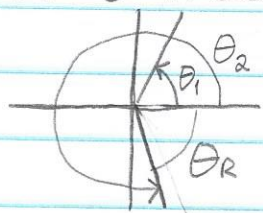
$$\theta_2 = 360^\circ - \theta_R = 360^\circ - 19.8^\circ = 340.2^\circ$$

$$(b) \cos \theta = 0.1275$$

$$\theta = \cos^{-1}(0.1275)$$

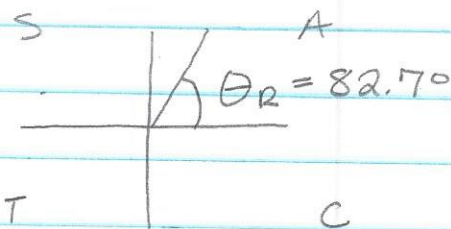
$$\theta = 82.7^\circ$$

$\cos \theta > 0$ in QI and QIV



$$\theta_1 = \theta_R$$

$$\theta_2 = 360^\circ - \theta_R = 360^\circ - 82.7^\circ = 277.3^\circ$$



$$(c) \tan \theta = 0.3849$$

$$\theta = \tan^{-1}(0.3849)$$

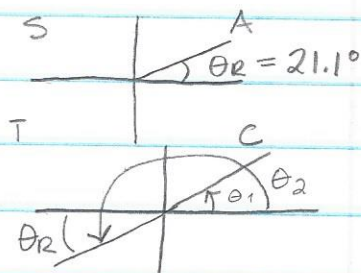
$$\theta = 21.1^\circ$$

$\tan \theta > 0$ in QI and QIII

$$\theta_1 = \theta_R$$

$$= 21.1^\circ$$

$$\theta_2 = 180^\circ + \theta_R = 180^\circ + 21.1^\circ = 201.1^\circ$$



$$(d) \cot \theta = -3.3014$$

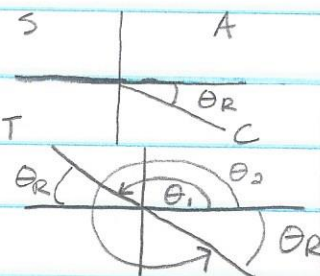
$$\theta = \tan^{-1}\left(\frac{1}{-3.3014}\right)$$

$$\theta = -16.9^\circ$$

$\cot \theta < 0$ in QII and QIV

$$\theta_1 = 180^\circ - \theta_R = 180^\circ - 16.9^\circ = 163.1^\circ$$

$$\theta_2 = 360^\circ - \theta_R = 360^\circ - 16.9^\circ = 343.1^\circ$$

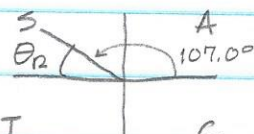


$$(e) \sec \theta = -3.4128$$

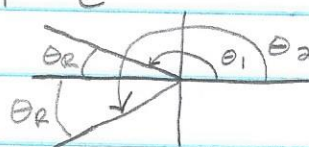
$$\theta = \cos^{-1}\left(\frac{1}{-3.4128}\right)$$

$$\theta = 107.0^\circ$$

$\sec \theta < 0$ in QII and QIII



$$\theta_R = 180^\circ - 107.0^\circ = 73.0^\circ$$



$$\theta_1 = 180^\circ - \theta_R = 180^\circ - 73.0^\circ = \boxed{107.0^\circ}$$

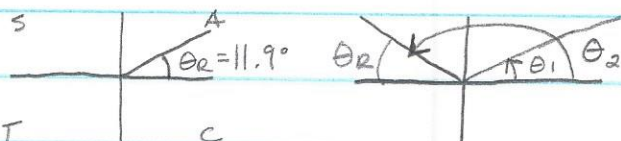
$$\theta_2 = 180^\circ + \theta_R = 180^\circ + 73.0^\circ = \boxed{253.0^\circ}$$

$$(f) \csc \theta = 4.8528$$

$$\theta = \sin^{-1} \left(\frac{1}{4.8528} \right)$$

$$\theta = 11.9^\circ$$

$\csc \theta > 0$ in QI and QII



$$\theta_1 = \theta_R = \boxed{11.9^\circ}$$

$$\theta_2 = 180^\circ - \theta_R = \boxed{168.1^\circ}$$

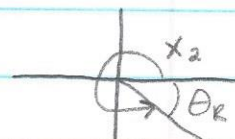
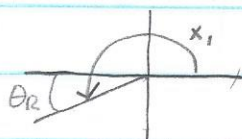
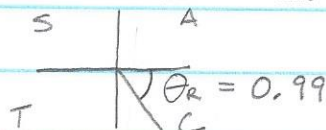
Ex.4 Find the general solution: $x = \boxed{} + \boxed{} \pi n$.

$$(a) \sin x = -0.8358$$

$$x = \sin^{-1}(-0.8358)$$

$$x = -0.99$$

$\sin x < 0$ in QIII and QIV



$$x_1 = \pi + \theta_R = \pi + 0.99 = \boxed{4.13 + 2\pi n}$$

$$x_2 = 2\pi - \theta_R = 2\pi - 0.99 = \boxed{5.29 + 2\pi n}$$

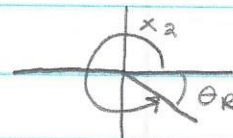
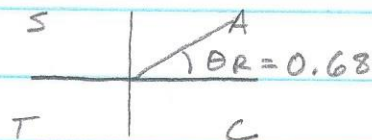
$$\begin{array}{r} 1.5708 \\ 3.1416 + 0 \\ 4.7124 \\ -1.5708 \end{array}$$

$$(b) \cos x = 0.7807$$

$$x = \cos^{-1}(0.7807)$$

$$x = 0.68$$

$\cos x > 0$ in QI and QIV



$$x_1 = \theta_R = \boxed{0.68 + 2\pi n}$$

$$x_2 = 2\pi - \theta_R = 2\pi - 0.68 = \boxed{5.61 + 2\pi n}$$

Note: Only one solution for $\tan x$ and $\cot x$ because QI/QIII and QII/QIV are opposite each other

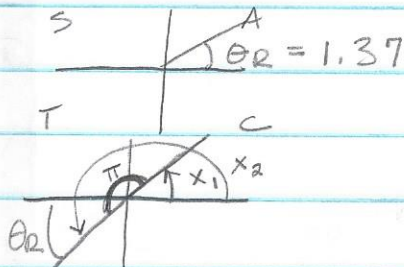
(c) $\tan x = 4.8743$

$$x = \tan^{-1}(4.8743)$$

$$x = 1.37$$

$\tan x > 0$ in QI and QIII

$$x = 1.37 + \pi n$$



(d) $\cot x = -2.8778$

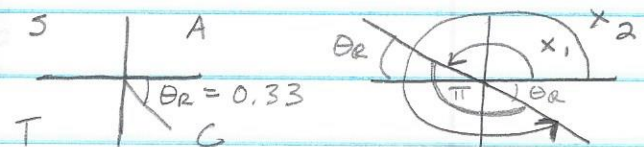
$$x = \tan^{-1}\left(\frac{1}{-2.8778}\right)$$

$$x = -0.33$$

$\cot x < 0$ in QII and QIV

$$x = \pi - \theta_R = \pi - 0.33$$

$$x = 2.81 + \pi n$$



(e) $\sec x = -3.0499$

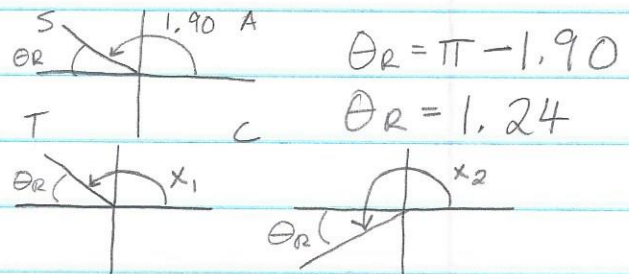
$$x = \cos^{-1}\left(\frac{1}{-3.0499}\right)$$

$$x = 1.90$$

$\sec x < 0$ in QII and QIII

$$x_1 =$$

$$x_2 = \pi + \theta_R = \pi + 1.24 = 4.38 + 2\pi n$$



(f) $\csc x = 2.1412$

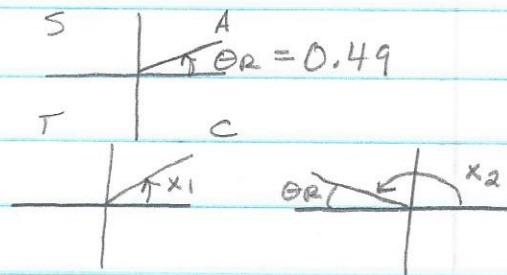
$$x = \sin^{-1}\left(\frac{1}{2.1412}\right)$$

$$x = 0.49$$

$\csc x > 0$ in QI and QII

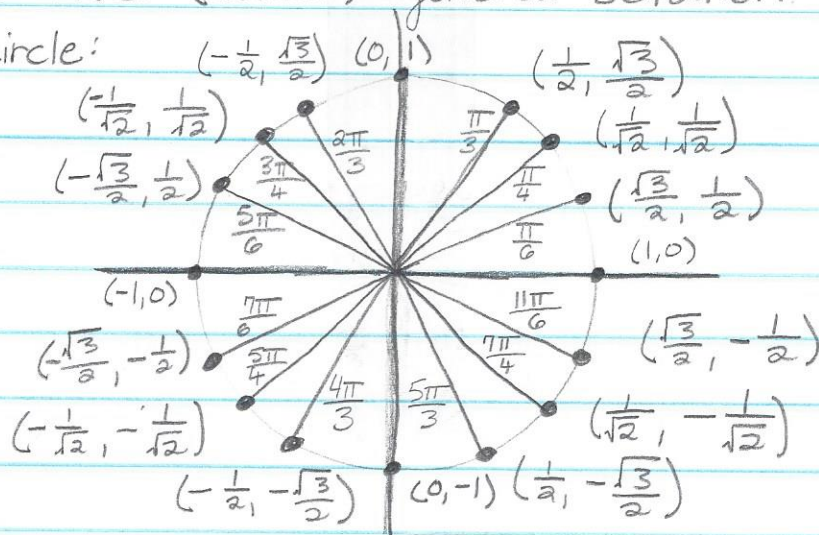
$$x_1 =$$

$$x_2 = \pi - \theta_R = \pi - 0.49 = 2.66 + 2\pi n$$



Ex5 Find the (exact) general solution: $x = \square + \square \pi n$

Unit circle:



(a) $\csc x = \sqrt{2} \Rightarrow x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$$x_1 = \frac{\pi}{4} + 2\pi n$$

$$x_2 = \frac{3\pi}{4} + 2\pi n$$

(b) $\cot x = -\frac{1}{\sqrt{3}} \Rightarrow x = \tan^{-1}(-\sqrt{3}) = \tan^{-1}\left(-\frac{\sqrt{3}/2}{1/2}\right)$

$$x = \frac{2\pi}{3} + \pi n$$

(c) $\sin x = -\frac{\sqrt{2}}{2} \Rightarrow x = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$$x_1 = \frac{5\pi}{4} + 2\pi n$$

$$x_2 = \frac{7\pi}{4} + 2\pi n$$

(d) $\sec x = 2 \Rightarrow x = \cos^{-1}\left(\frac{1}{2}\right)$

$$x_1 = \frac{\pi}{3} + 2\pi n$$

$$x_2 = \frac{5\pi}{3} + 2\pi n$$

(e) $\tan x = \frac{1}{\sqrt{3}} \Rightarrow x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{1/2}{\sqrt{3}/2}\right)$

$$x = \frac{\pi}{6} + \pi n$$

(f) $\cos x = -1 \Rightarrow x = \cos^{-1}(-1)$

$$x = \pi + 2\pi n$$