

Lesson 29: Simplifying Algebraic Expressions

* Factoring!

$$\text{Factor } x^4 y^6 + x^5 y^2 = x^4 y^2 (y^4 + x).$$

What did we do?

1. Found the smallest exponent on each and pulled those out.

2. Subtracted the amount you factored out from each exponent.

$$\begin{aligned} \text{i.e. } x^4 y^6 + x^5 y^2 &= x^4 y^2 (x^{4-4} y^{6-2} + x^{5-4} y^{2-2}) \\ &= x^4 y^2 (x^0 y^4 + x y^0) \\ &= x^4 y^2 (1 \cdot y^4 + x \cdot 1) \\ &= x^4 y^2 (y^4 + x) \end{aligned}$$

Ex.1 Factor and simplify as much as possible.

$$\begin{aligned} \text{(a) } (6x+12)^3 (5x+2)^5 + (6x+12)^4 (5x+2)^4 & \\ &= (6x+12)^3 (5x+2)^4 [(6x+12)^{3-3} (5x+2)^{5-4} + (6x+12)^{4-3} (5x+2)^{4-4}] \\ &= (6x+12)^3 (5x+2)^4 [(5x+2) + (6x+12)] \\ &= (6x+12)^3 (5x+2)^4 (11x+14) \end{aligned}$$

$$\begin{aligned} \text{(b) } 6(4x^2+13)^6 (x-12)^5 + 72x(4x^2+13)^5 (x-12)^6 & \\ &= 6(4x^2+13)^5 (x-12)^5 [(4x^2+13)^{6-5} (x-12)^{5-5} + 12x(4x^2+13)^{5-5} (x-12)^{6-5}] \\ &= 6(4x^2+13)^5 (x-12)^5 [(4x^2+13) + 12x(x-12)] \\ &= 6(4x^2+13)^5 (x-12)^5 [4x^2+13 + 12x^2 - 144x] \\ &= 6(4x^2+13)^5 (x-12)^5 (16x^2 - 144x + 13) \end{aligned}$$

$$\begin{aligned}
 - (c) & 72(8x-16)^8(2x+16)^4 + 8(8x-16)^9(2x+16)^3 \\
 & = 8(8x-16)^8(2x+16)^3 [9(8x-16)^{8-8}(2x+16)^{4-3} + (8x-16)^{9-8}(2x+16)^{3-3}] \\
 & = 8(8x-16)^8(2x+16)^3 [9(2x+16) + (8x-16)] \\
 & = 8(8x-16)^8(2x+16)^3(26x+128)
 \end{aligned}$$

$$\begin{aligned}
 - (d) & (2x+3)^3(5x-1)^4 + (2x+3)^2(5x-1)^5 \\
 & = (2x+3)^2(5x-1)^4 [(2x+3) + (5x-1)] \\
 & = (2x+3)^2(5x-1)^4(7x+2)
 \end{aligned}$$

Ex.2 Factor and simplify.

$$\begin{aligned}
 (a) & (x+3)^{1/2}(2x+1)^{2/5} - 2(x+3)^{3/2}(2x+1)^{2/5} \\
 & = (x+3)^{1/2}(2x+1)^{2/5} [(x+3)^{\frac{1}{2}-\frac{3}{2}}(2x+1)^{\frac{2}{5}-\frac{2}{5}} - 2(x+3)^{\frac{3}{2}-\frac{1}{2}}(2x+1)^{\frac{2}{5}-\frac{2}{5}}] \\
 & = (x+3)^{1/2}(2x+1)^{2/5} [(2x+1) - 2(x+3)] \\
 & = -5(x+3)^{1/2}(2x+1)^{2/5}
 \end{aligned}$$

$$\begin{aligned}
 (b) & (5x-3)^{3/2}(x+9)^{1/5} + (5x-3)^{1/2}(x+9)^{6/5} \\
 & = (5x-3)^{1/2}(x+9)^{1/5} [(5x-3) + (x+9)] \\
 & = (5x-3)^{1/2}(x+9)^{1/5}(6x+6)
 \end{aligned}$$

Ex.3 Factor and simplify. Do not leave negative exponents.

$$\begin{aligned}
 (a) & (2x+5)^3(x-4)^{-4} + (2x+5)^4(x-4)^{-5} \quad \text{Note: } -5 < -4 \\
 & = (2x+5)^3(x-4)^{-5} [(2x+5)^{3-3}(x-4)^{-4-(-5)} + (2x+5)^{4-3}(x-4)^{-5-(-5)}] \\
 & = (2x+5)^3(x-4)^{-5} [(x-4) + (2x+5)] \\
 & = (2x+5)^3(x-4)^{-5}(3x+1) \\
 & = \frac{(2x+5)^3(3x+1)}{(x-4)^5}
 \end{aligned}$$

$$\begin{aligned}
 (b) & -10(x-1)^6(5x+10)^{-3} + 6(x-1)^5(5x+10)^{-2} \quad \text{Note: } -3 < -2 \\
 & = 2(x-1)^5(5x+10)^{-3} [-5(x-1) + 3(5x+10)] \\
 & = 2(x-1)^5(5x+10)^{-3} (10x+35) \\
 & = \frac{2(x-1)^5(10x+35)}{(5x+10)^3}
 \end{aligned}$$

$$\begin{aligned}
 (c) & -10(x^2+7)^{-6}(8-2x)^{-7} + 14(x^2+7)^{-5}(8-2x)^{-8} \\
 & = 2(x^2+7)^{-6}(8-2x)^{-8} [-5(8-2x) + 7(x^2+7)] \\
 & = 2(x^2+7)^{-6}(8-2x)^{-8} (7x^2+10x+9) \\
 & = \frac{2(7x^2+10x+9)}{(x^2+7)^6(8-2x)^8}
 \end{aligned}$$

$$\begin{aligned}
 - (d) & 12(5x-2)^{-3}(3x+3)^3 - 15(5x-2)^{-4}(3x+3)^4 \\
 & = 3(5x-2)^{-4}(3x+3)^3 [4(5x-2) - 5(3x+3)] \\
 & = 3(5x-2)^{-4}(3x+3)^3 (5x-23) \\
 & = \frac{3(3x+3)^3(5x-23)}{(5x-2)^4}
 \end{aligned}$$

Ex.4 Factor and simplify. No negative exponents.

$$\begin{aligned}
 (a) & (x+4)^{6/5}(4x+10)^{1/2} + (x+4)^{11/5}(4x+10)^{-1/2} \\
 & = (x+4)^{6/5}(4x+10)^{-1/2} [(x+4)^{6/5-6/5}(4x+10)^{1/2-(-1/2)} + (x+4)^{11/5-6/5}(4x+10)^{-1/2+1/2}] \\
 & = (x+4)^{6/5}(4x+10)^{-1/2} [(4x+10) + (x+4)] \\
 & = (x+4)^{6/5}(4x+10)^{-1/2} (5x+14) \\
 & = \frac{(x+4)^{6/5}(5x+14)}{(4x+10)^{1/2}}
 \end{aligned}$$

$$\begin{aligned}
 - (b) & (10x^2+2)^{3/4}(10x+2)^{1/3} + (10x^2+2)^{7/4}(10x+2)^{-2/3} \\
 & = (10x^2+2)^{3/4}(10x+2)^{-2/3} [(10x+2) + (10x^2+2)] \\
 & = \frac{(10x^2+2)^{3/4}(10x^2+10x+4)}{(10x+2)^{2/3}}
 \end{aligned}$$

$$\begin{aligned}
 & - (c) (7x-8)^{-7/4} (5x+5)^{-1/6} + (7x-8)^{-3/4} (5x+5)^{-7/6} \\
 & = (7x-8)^{-7/4} (5x+5)^{-7/6} [(5x+5) + (7x+8)] \\
 & = \frac{12x-3}{(7x-8)^{7/4} (5x+5)^{7/6}}
 \end{aligned}$$

Ex.5 Factor and simplify. No negative exponents.

$$\begin{aligned}
 (a) \frac{2x(x^2+2)^3 - 4x^2(x^2+2)^2}{(x^2+2)^7} &= \frac{2x(x^2+2)^2 [(x^2+2) - 2x]}{(x^2+2)^7} \\
 &= \frac{2x(x^2 - 2x + 2)}{(x^2+2)^7}
 \end{aligned}$$

$$\begin{aligned}
 - (b) \frac{(4x+5)^{1/3} (x+1)^2 - (4x+5)^{-2/3} (x+1)^3}{4x+5} &= \frac{(4x+5)^{-2/3} (x+1) [(4x+5) - (x+1)]}{(4x+5)} \\
 &= \frac{(x+1)(3x+4)}{(4x+5)^{5/3}}
 \end{aligned}$$

$$\begin{aligned}
 - (c) \frac{2(x+3)^4 (7x+1)^3 - 2(x+3)^3 (7x+1)^4}{(x+3)^6} &= \frac{2(x+3)^3 (7x+1)^3 [(x+3) - (7x+1)]}{(x+3)^6} \\
 &= \frac{2(7x+1)^3 (-6x+2)}{(x+3)^3}
 \end{aligned}$$

$$= (d) \frac{2x-7}{(4x+1)^{1/2} (x+13)} \quad \checkmark$$

