Lesson 30: Systems of Equations

Ex. 1 Solve the systems using substitution or elimination. Classify them.

(a) $\begin{cases} 2x+3y=5 \\ x-y=1 \end{cases}$ Substitution: $x-y=1 \Rightarrow x=y+1$

Then $2(y+1)+3y=5$
$2y+2+3y=5$
$5y+2=5$
$5y=3$
$y=3/5$

$x=\frac{3}{5}+1$
$x=\frac{8}{5}$

Solution: $\boxed{x=\frac{8}{5},\ y=\frac{3}{5}}$

Def Consistent Independent: One solution (lines intersect in one place)

(b) $\begin{cases} 3x+y=6 \\ 6x+2y=12 \end{cases}$ Elimination:

$$\begin{array}{r} 6x+2y=12 \\ -2(3x+y=6) \\ \hline 0=0 \end{array}$$

Solution: $0=0$

Def Consistent Dependent: Infinite solution (the lines are the same)

(c) $\begin{cases} 3x+y=6 \\ 6x+2y=20 \end{cases}$ Elimination:

$$\begin{array}{r} 6x+2y=20 \\ -2(3x+y=6) \\ \hline 0=8 \end{array}$$

Solution: $0=8$

Def. Inconsistent: No solution (lines are parallel and never intersect)

Ex. 2 Solve the system $\begin{cases} x^2+y^2=4 \\ y-x=1 \end{cases}$ (circle with radius 2)

$y=x+1 \Rightarrow x^2+(x+1)^2=4$
$x^2+x^2+2x+1=4$
$2x^2+2x-3=0$

$x=\frac{-2\pm\sqrt{4-4(2)(-3)}}{2(2)}$
$x=\frac{-2\pm\sqrt{28}}{4}$
$x=\frac{-1\pm\sqrt{7}}{2}$

$x = \frac{-1+\sqrt{7}}{2}$ $\qquad\qquad$ $x = \frac{-1-\sqrt{7}}{2}$

$y = \left(\frac{-1+\sqrt{7}}{2}\right) + 1$ $\qquad\qquad$ $y = \frac{-1-\sqrt{7}}{2} + 1$

$y = \frac{1+\sqrt{7}}{2}$ $\qquad\qquad$ $y = \frac{1-\sqrt{7}}{2}$

Solutions: $\left(\frac{-1+\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right), \left(\frac{-1-\sqrt{7}}{2}, \frac{1-\sqrt{7}}{2}\right)$

Note: 3 possibilities for non-linear systems:

(1) $b^2 - 4ac > 0$ $\quad$ 2 solutions $\quad$ (line passes through circle)

(2) $b^2 - 4ac = 0$ $\quad$ 1 solution $\quad$ (line is tangent to circle)

(3) $b^2 - 4ac < 0$ $\quad$ no solution $\quad$ (they do not intersect)

Ex.3 Find all values of k so that the system has

(a) One Solution

(b) Two solutions

(c) No solution

$$\begin{cases} x^2 + y^2 = 2 \\ y = x + k \end{cases}$$

$x^2 + (x+k)^2 = 2$

$x^2 + x^2 + 2kx + k^2 = 2$

$2x^2 + 2kx + k^2 - 2 = 0$

$a = 2,\ b = 2k,\ c = k^2 - 2 \Rightarrow b^2 - 4ac = (2k)^2 - 4(2)(k^2-2)$

$= -4k^2 + 16$

$-4k^2 + 16 = 0$

$16 = 4k^2$

$4 = k^2$

$\pm 2 = k$

(a) $x = -2, 2$

(b) $(-2, 2)$

(c) $(-\infty, -2) \cup (2, \infty)$

1 sol'n $\quad$ 1 sol'n

$- \quad 0 \quad + \quad 0 \quad -$ $\qquad$ $-4k^2+16$

no sol'n $\ -2\ $ 2 sol'n $\ 2\ $ no sol'n $\quad k$

Lesson 30 Examples

1. A gateship sets out for a test ride on the ocean going against the current (upstream) 7 miles and then turning around and returning to Atlantis. The trip against the current (upstream) requires 30 minutes and the trip with the current (downstream) requires 16 minutes. Find the speed of the gateship in still water and the speed of the current in miles per hour (mi/h).

Let x = speed of gateship
y = speed of current.

Quantity = rate × time

Upstream: $7 = (x-y)\left(\frac{1}{2}\right) \Rightarrow 14 = x - y$

Downstream: $7 = (x+y)\left(\frac{16}{60}\right) \Rightarrow 26.25 = x + y$

$40.25 = 2x$

$20.125 = x$

$14 = 20 - y$

$\Rightarrow y = 20 - 14 = 6$

$\boxed{\begin{array}{l} x = 20 \text{ mph} \\ y = 6 \text{ mph} \end{array}}$

2. Michael Scott works a total of 50 hours per week at two jobs. His total pay is \$480.25 before taxes. He works 35 hours at Dunder-Mifflin and 15 hours at the Lipephedrine Diet Pill Company (LDPC). If his hourly rate at Dunder-Mifflin is \$2.65 more than his hourly rate at LDPC, find Michael's pay per hour at each job.

Let x = rate at DM
y = rate at LDPC.

Quantity = rate × time

DM: $35x$

LDPC: $15y$

Total: $480.25 = 35x + 15y$

Relation: $x = y + 2.65$

$480.25 = 35(y + 2.65) + 15y$

$480.25 = 50y + 92.75$

$387.50 = 50y$

$7.75 = y$

$x = 7.75 + 2.65$

$x = 10.40$

$\boxed{\begin{array}{l} x = \$10.40 \\ y = \$7.75 \end{array}}$

3. Ronan Dex and Teyla Emmagan work together evacuating a village from the Wraith. The mission required 7.5 hours to complete. In an identical village on the same planet, Ronan and Teyla worked together for 4.6 hours before Ronan had to leave to rescue Rodney. Teyla completed the village evacuation after 6.5 more hours. Working alone, how long would it take Ronan and Teyla to evacuate a village of this size?

Let x = hours for Ronan $\Rightarrow$ rate for Ronan $= \frac{1}{x}$ $\frac{\text{village}}{\text{hour}}$

y = hours for Teyla   rate for Teyla $= \frac{1}{y}$ $\frac{\text{village}}{\text{hour}}$

Quantity = rate x time

Together: $1 \text{ village} = \left(\frac{1}{x} + \frac{1}{y}\right)(7.5) \Rightarrow \frac{1}{7.5} = \frac{1}{x} + \frac{1}{y}$

Just Teyla: $\left(1 - \frac{4.6}{7.5}\right) = \frac{29}{75} = \frac{1}{y}(6.5) \Rightarrow \frac{58}{975} = \frac{1}{y}$

portion done together

$y = 16.8$

$\frac{1}{7.5} = \frac{1}{16.8} + \frac{1}{x}$

$0.074 = \frac{1}{x} \Rightarrow x = 13.5$

$\boxed{\begin{array}{l} y = 16.8 \text{ hours} \\ x = 13.5 \text{ hours} \end{array}}$

4. What quantities of Trinium 98.8% pure and 84.8% pure must be mixed together to give 17.5 grams of Trinium 89.9% pure?

x = grams of 98.8% $\Rightarrow 17.5 = x + y$

y = grams of 84.8% $17.5 - y = x$

amount of pure silver:

$(17.5)(.899) = .988x + .848y$

$15.7325 = .988(17.5 - y) + .848y$

$15.7325 = 17.29 - 0.14y$

$0.14y = 1.5575$

$y = 11.125$

$\boxed{\begin{array}{l} x = 6.4g \\ y = 11.1g \end{array}}$

$x = 17.5 - 11.125$

$x = 6.375$