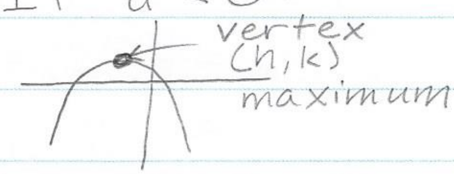
Lesson 8: Quadratic Functions (Parabolas)

General Form: $f(x) = ax^2 + bx + c$

If $a > 0$:

If $a < 0$:



Important Info!

Vertex: (h, k), $h = -\frac{b}{2a}$

$k = f(h)$

Zeros (x-intercepts): Where $f(x) = 0$.

Solve $ax^2 + bx + c = 0$.

- Factor.
- Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$f(x) = x^2 + 1$

$\leftarrow$ (if $b^2 - 4ac < 0$ (neg), no roots)

y-intercepts: Where $x = 0$, so find $f(0)$.

$(0, y)$

Note the minus sign!

Standard Form: $y = a(x-h)^2 + k$

$(h, k) = \text{vertex}$

Ex1 Find $y = x^2 - 6x + 5$ in standard form.

Complete the square. OR Find vertex (h, k).

$y = (x^2 - 6x + \underline{(-3)^2}) + 5 - \underline{(-3)^2}$

$y = (x^2 - 6x + 9) + 5 - 9$

$\boxed{y = (x-3)^2 - 4}$

$h = -\frac{b}{2a} = -\frac{(-6)}{2(1)}$

$= \frac{6}{2} = 3$

$k = f(3) = 3^2 - 6(3) + 5$

$= 9 - 18 + 5 = -4$

$\boxed{y = 1(x-3)^2 - 4}$

Ex.2 Find $y=-2x^2-8x+9$ in standard form.

$a=-2$
$b=-8$
$c=9$

$$h=-\frac{b}{2a}=-\frac{(-8)}{2(-2)}$$
$$=\frac{8}{-4}=-2$$
$$k=f(-2)=-2(-2)^2-8(-2)+9$$
$$=-2\cdot 4+16+9=17$$
$$y=-2(x-(-2))^2+17$$
$$\boxed{y=-2(x+2)^2+17}$$

Ex.3 Find (a) $y=x^2+4x+1$ in standard form.
(b) the vertex (max or min?).
(c) the zeros.
(d) the y-intercept.

(a) $y=(x^2+4x+\underline{(\frac{4}{2})^2})+1-\underline{(\frac{4}{2})^2}$
$y=(x^2+4x+4)+1-4$
$y=(x+2)^2-3$

(b) Vertex: $(-2,-3)$ min

$$h=-\frac{b}{2a}=-\frac{4}{2}=-2$$
$$k=f(h)=f(-2)=4-8+1=-3$$

(c) $x^2+4x+1=0$

$$x=\frac{-4\pm\sqrt{16-4}}{2}$$
$$x=\frac{-4\pm\sqrt{12}}{2}$$
$$x=\frac{-4\pm 2\sqrt{3}}{2}$$
$$x=-2\pm\sqrt{3}$$

$$\boxed{\begin{aligned}x&=-2+\sqrt{3}\\x&=-2-\sqrt{3}\end{aligned}}$$

(d) $y=0^2+4(0)+1=1$

$\boxed{(0,1)}$

Ex.4 Find the standard equation of a parabola with vertex (1,5) passing through (-4,10).

Since $(h,k) = (1,5)$, $y = a(x-1)^2 + 5$.

(-4,10) lies on the graph, so solve for a:

$$10 = a(-4-1)^2 + 5$$

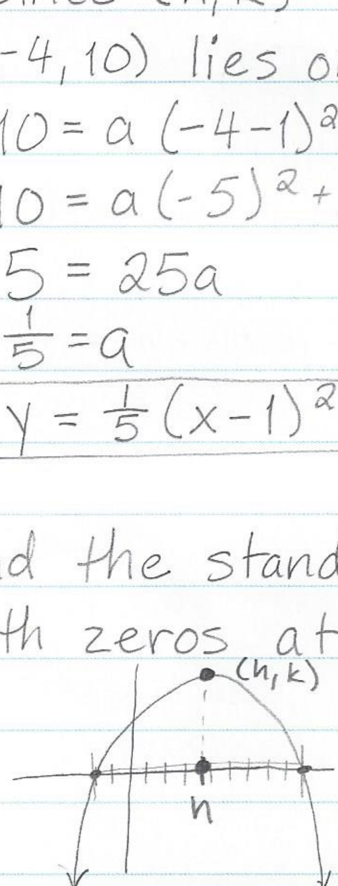
$$10 = a(-5)^2 + 5$$

$$5 = 25a$$

$$\frac{1}{5} = a$$

$$\Rightarrow \boxed{y = \frac{1}{5}(x-1)^2 + 5}$$

Ex.5 Find the standard equation of a parabola with zeros at $x=-2$ and $x=9$ passing through (0,9).



$$h = \frac{-2+9}{2} = \frac{7}{2}$$

(-2,0), (9,0) and (0,9) lie on the parabola.

$$y = a\left(x - \frac{7}{2}\right)^2 + k$$

At (9,0): $0 = a\left(9 - \frac{7}{2}\right)^2 + k$

$$0 = a\left(\frac{11}{2}\right)^2 + k$$

$$-a\left(\frac{121}{4}\right) = k$$

At (0,9): $9 = a\left(-\frac{7}{2}\right)^2 + k$

$$9 - a\left(\frac{49}{4}\right) = k$$

Then equate $9 - \frac{49}{4}a = -\frac{121}{4}a$

$$9 = -\frac{72}{4}a = -18a$$

$$-\frac{1}{2} = a$$

Plug in here to find k.

$$-\left(-\frac{1}{2}\right)\left(\frac{121}{4}\right) = k$$

$$\frac{1}{2} \cdot \frac{121}{4} = k$$

$$\frac{121}{8} = k$$

Then: $\boxed{y = -\frac{1}{2}\left(x - \frac{7}{2}\right)^2 + \frac{121}{8}}$