Exam 2 Review

Lessons 11 - 12: The Chain Rule and Derivative of Natural Log

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) * g'(x)$$

OR

$$\frac{d}{dx}[Out(In)] = Out'(In) * In'$$

The derivative of the ***outside*** (with the ***inside*** plugged back in) times the derivative of the ***inside***.

- **Special Case:** $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x) * e^{f(x)}$ (Note: the exponent does NOT change.)
- $\boldsymbol{\frac{d}{dx}(\ln(x)) = \frac{1}{x}}$

- Use rules for logs and exponents to potentially make computing derivatives easier.

$\ln(xy) = \ln(x) + \ln(y)$	$e^x e^y = e^{x+y}$	$x^a x^b = x^{a+b}$
$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$	$\frac{e^x}{e^y} = e^{x-y}$	$\frac{x^a}{x^b} = x^{a-b}$
$\ln(x^y) = y\ln(x)$	$\frac{1}{e^x} = e^{-x}$	$\boldsymbol{\sqrt[b]{x^a} = x^{\frac{a}{b}}}$
$x^{-a} = \frac{1}{x^a}$	$(e^x)^y = e^{xy}$	$(x^a)^b = x^{ab}$

Lesson 13: Higher Order Derivatives

- $f^{(n)}(x)$, $\frac{d^n y}{dx^n}$, $y^{(n)}$ all mean to take the derivative n times.
- If $s(t)$ is a position function, $s'(t) = v(t)$ is velocity, and $s''(t) = v'(t) = a(t)$ is acceleration.

Lesson 14: Implicit Differentiation

Implicit Differentiation

$$\frac{d}{dx}[f(y)] = \frac{dy}{dx} * f'(y)$$

- Use when y is not explicitly solved for. For instance: $y^2 = e^x$, $\cos(xy) = x$, etc.
- Basically, anytime we "touch/change" a y, we need to multiply $\frac{dy}{dx}$ or y' to the term, so take the derivative of what you see and put the $\frac{dy}{dx}$ or y' next to it.
- For example,

  $\frac{d}{dx}(e^y) = \frac{dy}{dx} * e^y$

  $\frac{d}{dx}(\ln(y)) = \frac{dy}{dx} * \frac{1}{y}$

  $\frac{d}{dx}(x * y) = 1 * y + x * \frac{dy}{dx}$ (product rule)

  $\frac{d}{dx}(e^{xy}) = \left(y + x\frac{dy}{dx}\right)e^{xy}$ (chain rule and product rule)

  $\frac{d}{dx}(y^2) = \frac{dy}{dx}\, 2y$
- Your $\frac{dy}{dx}$ or y' will **not** appear inside a trig function, denominator, or exponent.

Lessons 15-16: Related Rates

- Determine what formula you need to use.
- Plug in any ***constant*** quantities, i.e. quantities that do NOT change with ***time***.
- Take derivative of both sides with respect to ***time***.
- **Pay attention to units!**

Shape	Perimeter P or Surface Area S	Area A or Volume V
Rectangle with sides l and w	$P = 2l + 2w$	$A = lw$
Square with side x	$P = 4x$	$A = x^2$
Circle with radius r	**(Circumference C)** $C = 2\pi r$	$A = \pi r^2$
Cube with side x	$S = 6x^2$	$V = x^3$

Trapezoid	(with sides a, b, c, d) $P = a + b + c + d$	(with base b_1 and b_2 and height h) $A = \frac{1}{2}h\,(b_1 + b_2)$
Rectangular prism with sides l, w, h	$S = 2lw + 2lh + 2wh$	$V = lwh$
Triangle	(with sides $a,\ b$, and c) $P = a + b + c$	(with base b and height h) $A = \frac{1}{2}bh$

- For right triangles, we use the Pythagorean Theorem $x^2 + y^2 = D^2$.
- For angles, pick the trig identity that has a *constant* quantity and the *rate* of the other quantity given.

Lessons 17-18: Relative Extrema, Critical Numbers, and the First Derivative Test

- We can use the *first derivative* to find out information about the *function* itself.
- **Critical Numbers:** where the derivative of a function ($f'(x)$) is $= \mathbf{0}$ or is ***undefined***.
- **A critical number must be in the *domain* of the function!**
- **Relative Extrema:** minimums or maximums of the entire function; function must be defined at the point; can have no relative extrema or can have multiple relative minimums or maximums

- $f(x)$ is **increasing** when $f'(x) > 0$
- $f(x)$ is **decreasing** when $f'(x) < 0$

- Relative extrema **only** occur at critical numbers, but not *all* critical numbers have relative extrema.
  - We *find* the critical numbers (always x-values) then *check* at each x coordinate to see if we have a relative minimum, a relative maximum or neither.

- **First Derivative Test:** Let c be a critical number for $f(x)$.
  - If $f'(x)$ goes from positive to negative at $x = c$, (i.e., $f(x)$ goes from increasing to decreasing),

    $\boldsymbol{f(c)}$ is a *relative maximum*.
  - If $f'(x)$ goes from negative to positive at $x = c$, (i.e., $f(x)$ goes from decreasing to increasing),

    $\boldsymbol{f(c)}$ is a *relative minimum*.
  - If $f'(x)$ does *not* change sign at $x = c$, $\boldsymbol{f(c)}$ is *neither a min nor a max*.
- **Use the original function to find the value of the minimum or maximum, i.e. $\boldsymbol{f(c)}$.**

- **Notation:** $x = c$ is *where* the relative min or max occurs; $f(c)$ is the *value* of the minimum or maximum. If they ask you to find the minimum or maximum, they want you to find the *value* of the function at that point.

Table of Derivatives

✳ **Derivatives of specific functions**

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\boxed{\frac{d}{dx}\ln(x) = \frac{1}{x}}$$

$$\frac{d}{dx}\big(\sin(x)\big) = \cos(x)$$

$$\frac{d}{dx}\big(\cos(x)\big) = -\sin(x)$$

$$\frac{d}{dx}\big(\tan(x)\big) = \sec^2(x)$$

$$\frac{d}{dx}\big(\cot(x)\big) = -\csc^2(x)$$

$$\frac{d}{dx}\big(\sec(x)\big) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\big(\csc(x)\big) = -\csc(x)\cot(x)$$

✳ **Differentiation Rules**

$$\frac{d}{dx}\big(c\,f(x)\big) = c\,f'(x)$$

$$\frac{d}{dx}\big(f(x) + g(x)\big) = f'(x) + g'(x)$$

$$\frac{d}{dx}\big(f(x) - g(x)\big) = f'(x) - g'(x)$$

$$\frac{d}{dx}(f(x)\,g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{\big(g(x)\big)^2} \quad \textbf{OR} \quad \frac{d}{dx}\left(\frac{Top}{Bottom}\right) = \frac{Top' * Bottom - Top * Bottom'}{(Bottom)^2}$$

✳ New Derivative Types

Chain Rule

$$\frac{d}{dx}[Out(In)] = Out'(In) * In'$$

Implicit Differentiation

$$\frac{d}{dx}[f(y)] = f'(y) * \frac{dy}{dx}$$

$$\frac{d}{dx}\big[e^{f(x)}\big] = f'(x)\;e^{f(x)}$$

$$\frac{d}{dt}[x + y] = \frac{dx}{dt} + \frac{dy}{dt}$$

$$\frac{d}{dx}\ln\big(f(x)\big) = \frac{f'(x)}{f(x)}$$