Exam 3 Review

Lessons 17-18: Relative Extrema, Critical Numbers, and First Derivative Test

(from exam 2 review - needed for curve sketching)

- **Critical Numbers:** where the derivative of a function is zero or undefined.
- A critical number must be in the domain of the function!
- Relative Extrema: minimums or maximums of the entire function; function must be defined there
- f(x) is increasing when f'(x) > 0
- f(x) is **decreasing** when f'(x) < 0
- **First Derivative Test:** Let *c* be a critical number for f(x).
 - If f'(x) goes from positive to negative at x = c, f(c) is a relative maximum.
 - If f'(x) goes from negative to positive at x = c, f(c) is a relative minimum.
 - If f'(x) does not change sign at x = c, f(c) is neither a relative min nor max.
- Notation: x = c is where the min/max occurs; f(c) is the value of the minimum or maximum. If they ask you to find the minimum or maximum, they want you to find the value of the function at that point.

Lesson 19: Concavity, Inflection Points, and the Second Derivative Test

- # f(x) is **concave up** when f''(x) > 0
- f'(x) is concave down when f''(x) < 0
- * An **inflection point** is where f(x) changes concavity. To find y-coordinate, plug into original function.
- * Second Derivative Test: Let c be a critical number for f(x) (i.e. f'(c) = 0).
 - If f''(c) > 0, f(c) is a relative minimum.
 - If f''(c) < 0, f(c) is a relative maximum.
 - If f''(c) = 0, the test is *inconclusive*, so use the first derivative test.

Lesson 20: Absolute Extrema on an Interval

How to find the absolute extrema of f(x) on a closed interval [a, b]:

- ① Find all critical numbers of f(x).
- ⁽²⁾ Evaluate f(x) at all critical numbers in [a, b], and at x = a and x = b.
- ③ Compare the f(x) values and identify the absolute min and max.

Lesson 21: Graphical Interpretations of Derivatives

When given a graph of f'(x), we can find where f(x) is/has the following:

- 1. critical numbers where f'(x) = 0 or does not exist, i.e. where f'(x) touches the *x*-axis.
- 2. increasing where f'(x) > 0, i.e where f'(x) is *above* the x-axis.

- 3. decreasing where f'(x) < 0, i.e where f'(x) is below the x-axis.
- 4. relative maximum where f'(x) goes from *positive* to *negative*.
- 5. relative minimum where f'(x) goes from *negative* to *positive*.
- 6. concave up where f''(x) > 0, i.e, where f'(x) is *increasing*
- 7. concave down where f''(x) < 0, i.e, where f'(x) is *decreasing*
- 8. inflection point f''(x) changes sign, i.e. f'(x) has a *horizontal* tangent line and switches from increasing to decreasing or decreasing to increasing

Lesson 22: Limits at Infinity

When evaluating a limit with $x \to \infty$ or $x \to -\infty$, take the highest x power in the numerator and denominator with their coefficients, cancel, and then evaluate.

- * Asymptotes (for f(x))
 - Vertical: Simplify f(x) and set the denominator equal to 0 and solve for x. (x = #)
 - Horizontal: Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to \infty} f(x)$. If one or both exist and equal some finite L, y = L is the HA.
 - Slant: Only occur when the degree of numerator is **exactly one more** than the degree of the denominator. Find by **polynomial division.** (Of the form y = mx + b.)

Lesson 23: Curve Sketching

- ✤ To sketch a curve, find the following:
 - 1. *x*-intercepts (when y = 0)
 - 2. *y*-intercept (when x = 0)
 - 3. Increasing/Decreasing Intervals (use *first derivative*)
 - 4. Concave Up/Concave Down Intervals (use *second derivative*)
 - 5. Inflection Points (use *second derivative*)
 - 6. Vertical Asymptotes
 - 7. Horizontal Asymptotes
 - 8. Slant Asymptotes
- * When making number lines for the first and second derivatives, be sure to mark where the function is **undefined**, check the sign (positive or negative) on both sides of the discontinuity. None of the intervals should include the *x*-values where f(x) is undefined.

Lessons 24-26: Optimization

Draw a picture!

- 1. Identify quantity to be optimized (maximized or minimized).
- 2. Identify the *constraint(s)*.
- 3. Solve the constraint for one of the variables. Substitute this into the equation from Step 1. (If you have more than one constraint, you will have to be a bit more creative.) This is your *objective function*.
- 4. Take the *derivative* with respect to the variable.
- 5. Set the derivative equal to zero and solve for the variable.
- 6. Find the desired quantity.

Note: In most cases, you will only get one plausible solution. However, you should be sure on the exam to use the $1^{st}DT$ or $2^{nd}DT$ to ensure what you found is, indeed, the minimum or the maximum.

Lessons 27-28: Antiderivatives and Indefinite Integration

- Rewrite the function, if necessary, to utilize the table of integration.
- We have yet to learn a method to "undo" the *chain*, *product*, or *quotient* rules.
- Once you have integrated, you can take the derivative to double check your integration.
- When given an initial value or initial values, you can use them to find the *particular solution* (i.e. you can solve for *C*).

Table of Derivatives

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^{n}) = n x^{n-1}, \quad n \neq 0$$

$$\frac{d}{dx}(e^{x}) = e^{x}$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\cos(x)) = \sec^{2}(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$$

$$\frac{d}{dx}(\cot(x)) = -\csc^{2}(x)$$

$$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}(cc(x)) = -cc(x)\cot(x)$$

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}(f(x)g(x)) = f'(g(x)) + f(x)g'(x)$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) + g'(x)$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) + g'(x)$$

Table of Integration

$$\int 0 \, dx = C$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$$

$$\int k \, dx = kx + C$$

$$\int e^x \, dx = e^x + C$$

$$\int \int e^x \, dx = e^x + C$$

$$\int \int e^x \, dx = \ln|x| + C$$

$$\int \cos(x) \, dx = \sin(x) + C$$

$$\int \sin(x) \, dx = -\cos(x) + C$$

$$\int \sec^2(x) \, dx = \tan(x) + C$$

$$\int \sec^2(x) \, dx = \tan(x) + C$$

$$\int \sec(x) \tan(x) \, dx = \sec(x) + C$$

$$\int \csc^2(x) \, dx = -\cot(x) + C$$

$$\int \csc^2(x) \, dx = -\cot(x) + C$$

$$\int \csc(x) \cot(x) \, dx = -\csc(x) + C$$

$$\int cf(x) \, dx = c \int f(x) \, dx$$

$$\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

We don't have a way to undo products or quotients except for the specific trig functions above. We also cannot undo the chain rule yet. To integrate products or

derivatives, rewrite them!

Formulas to know:

Shape	Perimeter <i>P</i> or Surface	Area A or Volume V
	Area <i>S</i>	
Rectangle with sides <i>l</i> and <i>w</i>	P = 2l + 2w	A = lw
Square with side <i>x</i>	P = 4x	$A = x^2$
	(Circumference <i>C</i>)	
Circle with radius r	$C = 2\pi r$	$A = \pi r^2$
	(with sides <i>a</i> , <i>b</i> , and <i>c</i>)	(with base b and height h)
Triangle	P = a + b + c	$A = \frac{1}{2}bh$
	(with sides <i>a</i> , <i>b</i> , <i>c</i> , <i>d</i>)	(with base b_1 and b_2 and
Trapezoid	P = a + b + c + d	height <i>h</i>)
		$A = \frac{1}{2} h (b_1 + b_2)$
Cube with side <i>x</i>	$S = 6x^2$	$V = x^3$
Rectangular prism with sides <i>l, w, h</i>	S = 2lw + 2lh + 2wh	V = lwh

Revenue = *units sold* * *price*

Profit = units sold * (price - cost)

Remember for optimization problems, drawing a picture will usually help you figure out what the equations are. In particular, pay attention to phrases like "open top box" or when a fence is "bounded on one side". Also, if any picture is given, make sure you use the variables as they appear on the picture.