Trig Facts

Let’s consider the right triangle below.

We usually remember the trig functions with “SOH CAH TOA” which means

- sine (S) is opposite (O) over hypotenuse (H)
- cosine (C) is adjacent (A) over hypotenuse (H)
- tangent (T) is opposite (O) over adjacent (A)
Now, we need to think about where a triangle falls on the coordinate plane.

If we look at the axes above, \( \theta \) travels counter clockwise from the positive x-axis towards the positive y-axis.

- If \( 0 < \theta < \frac{\pi}{2} \), \( \theta \) is in quadrant I, so x is positive and y is positive which means that all the trig functions will be positive.
- If \( \frac{\pi}{2} < \theta < \pi \), \( \theta \) is in quadrant II, so x is negative and y is positive which means that sine (and consequently, cosecant) is positive, but cosine and tangent (and consequently, secant and cotangent) are negative.
- If \( \pi < \theta < \frac{3\pi}{2} \), \( \theta \) is in quadrant III, so x is negative and y is negative which means that tangent (and consequently, cotangent) is positive, but sine and cosine (and consequently, cosecant and secant) are negative.
- If \( \frac{3\pi}{2} < \theta < 2\pi \), \( \theta \) is in quadrant IV, so x is positive and y is negative which means that cosine (and consequently, secant) is positive, but sine and tangent (and consequently, cosecant and cotangent) are negative.

To remember this, we can use the sentence “all students take calculus.”
Log and Exponent Rules

\[ \ln(xy) = \ln(x) + \ln(y) \quad e^x e^y = e^{x+y} \quad x^a x^b = x^{a+b} \]
\[ \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y) \quad \frac{e^x}{e^y} = e^{x-y} \quad \frac{x^a}{x^b} = x^{a-b} \]
\[ \ln(x^y) = y \ln(x) \quad \frac{1}{e^x} = e^{-x} \quad b\sqrt{x^a} = x^{\frac{a}{b}} \]
\[ x^{-a} = \frac{1}{x^a} \quad (e^x)^y = e^{xy} \quad (x^a)^b = x^{ab} \]

Recall that \( e^x = y \) means that \( \ln(y) = x \) and vice versa.