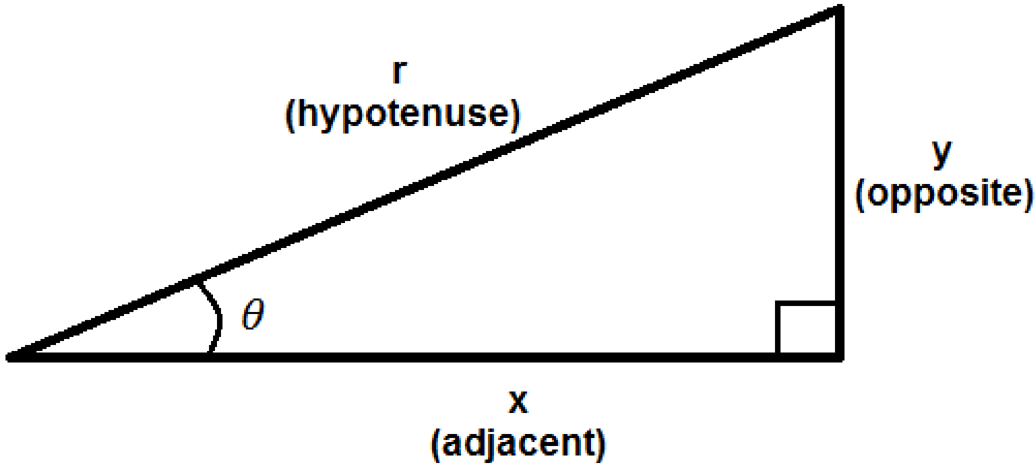
Trig Facts

Let's consider the right triangle below.



$$\cos(\theta) = \frac{x}{r} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x}$$

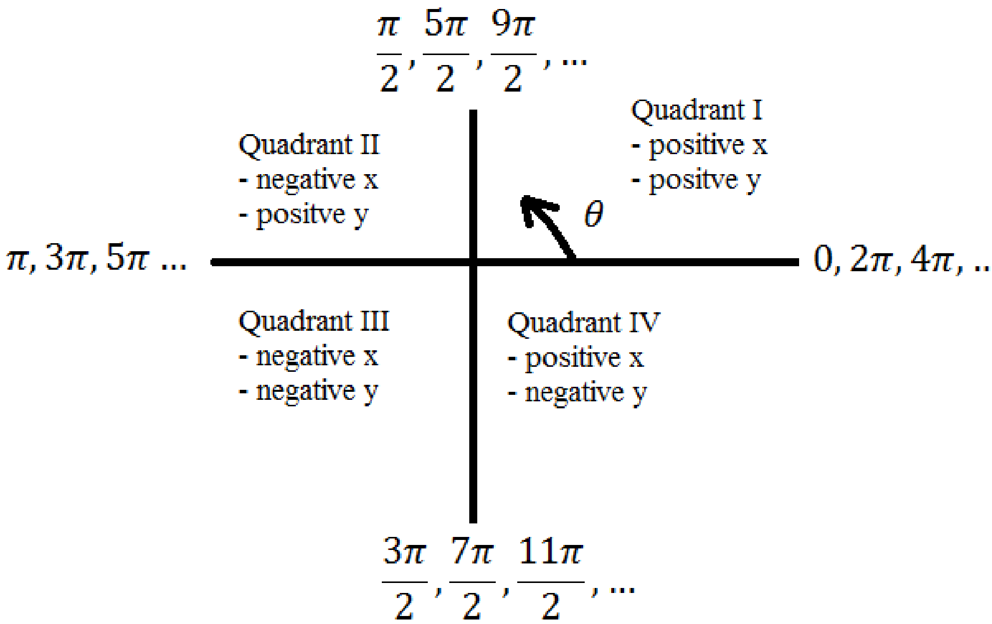
$$\sin(\theta) = \frac{y}{r} \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y}$$

$$\tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$$

We usually remember the trig functions with "SOH CAH TOA" which means

- sine (S) is opposite (O) over hypotenuse (H)
- cosine (C) is adjacent (A) over hypotenuse (H)
- tangent (T) is opposite (O) over adjacent (A)

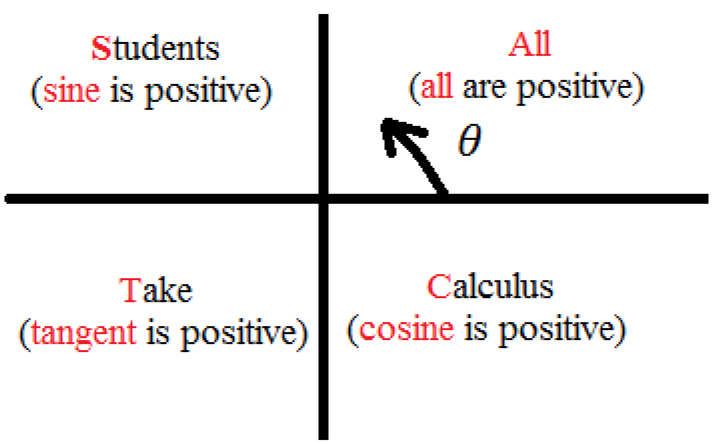
Now, we need to think about where a triangle falls on the coordinate plane.



If we look at the axes above, θ travels counter clockwise from the positive x-axis towards the positive y-axis.

- If $0 < \theta < \frac{\pi}{2}$, θ is in quadrant I, so x is positive and y is positive which means that *all* the trig functions will be positive.
- If $\frac{\pi}{2} < \theta < \pi$, θ is in quadrant II, so x is negative and y is positive which means that *sine* (and consequently, cosecant) is positive, but cosine and tangent (and consequently, secant and cotangent) are negative.
- If $\pi < \theta < \frac{3\pi}{2}$, θ is in quadrant III, so x is negative and y is negative which means that *tangent* (and consequently, cotangent) is positive, but sine and cosine (and consequently, cosecant and secant) are negative.
- If $\frac{3\pi}{2} < \theta < 2\pi$, θ is in quadrant IV, so x is positive and y is negative which means that *cosine* (and consequently, secant) is positive, but sine and tangent (and consequently, cosecant and cotangent) are negative.

To remember this, we can use the sentence "*a*ll *s*tudents *t*ake *c*alculus."



Log and Exponent Rules

$\ln(xy) = \ln(x) + \ln(y)$	$e^x e^y = e^{x+y}$	$x^a x^b = x^{a+b}$
$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$	$\frac{e^x}{e^y} = e^{x-y}$	$\frac{x^a}{x^b} = x^{a-b}$
$\ln(x^y) = y\ln(x)$	$\frac{1}{e^x} = e^{-x}$	$\sqrt[b]{x^a} = x^{\frac{a}{b}}$
$x^{-a} = \frac{1}{x^a}$	$(e^x)^y = e^{xy}$	$(x^a)^b = x^{ab}$

Recall that $e^x = y$ means that $\ln(y) = x$ and vice versa.