Problem 11:

Find the critical number(s) of $y = 6\cos(5x) + 15x$ on the interval $(0, \pi)$.

Solution:

Domain: $(-\infty,\infty)$

$$y' = 6 \frac{d}{dx} [\cos(5x)] + \frac{d}{dx} [15x]$$

Chain rule for $\frac{d}{dx} [\cos(5x)]$: Out = $\cos(x)$ In = 5x
Out' = $-\sin(x)$ In' = 5

$$\frac{d}{dx} [\cos(5x)] = Out'(In) * In'$$

$$= -\sin(5x) * 5$$

$$= -5\sin(5x)$$

$$y' = -30\sin(5x) + 15$$

$$0 = -30\sin(5x) + 15$$

$$30\sin(5x) = 15$$

$$\sin(5x) = \frac{1}{2}$$

 $5x = \arcsin\left(\frac{1}{2}\right) \text{ or } \sin^{-1}\left(\frac{1}{2}\right)$

To find $\arcsin\left(\frac{1}{2}\right)$, you can use the unit circle, or if you put your calculator in degrees, you will get 30° which is the same as $\frac{\pi}{6}$. (This way you won't be trying to figure out what the decimal radian angle is in terms of a fraction of pi.) But remember that there are two quadrants where sine is positive (quadrant I and II), AND if we add 2π to any angle, we get back to the same angle, so we have

$$5x = \frac{\pi}{6} + 2\pi n \qquad \text{and} \qquad 5x = \frac{5\pi}{6} + 2\pi n$$
$$x = \frac{1}{5} \left(\frac{\pi}{6} + 2\pi n\right) \qquad \text{and} \qquad x = \frac{1}{5} \left(\frac{5\pi}{6} + 2\pi n\right)$$
$$x = \frac{\pi}{30} + \frac{2\pi n}{5} \qquad \text{and} \qquad x = \frac{\pi}{6} + \frac{2\pi n}{5}$$

To find the critical numbers in the interval $(0, \pi)$, we need to keep adding $\frac{2\pi}{5}$ to each critical number until the angle exceeds π . Here, we would get:

$$\frac{\pi}{30} + \frac{2\pi}{5} = \frac{\pi}{30} + \frac{12\pi}{30} = \frac{13\pi}{30}$$
$$\frac{13\pi}{30} + \frac{2\pi}{5} = \frac{13\pi}{30} + \frac{12\pi}{30} = \frac{25\pi}{30} = \frac{5\pi}{6}$$
$$\frac{25\pi}{30} + \frac{2\pi}{5} = \frac{25\pi}{30} + \frac{12\pi}{30} = \frac{37\pi}{30} > \pi$$
$$\frac{\pi}{6}$$
$$\frac{\pi}{6} + \frac{2\pi}{5} = \frac{5\pi}{30} + \frac{12\pi}{30} = \frac{17\pi}{30}$$

 $\frac{17\pi}{30} + \frac{2\pi}{5} = \frac{17\pi}{30} + \frac{12\pi}{30} = \frac{29\pi}{30}$ $\frac{29\pi}{30} + \frac{2\pi}{5} = \frac{29\pi}{30} + \frac{12\pi}{30} = \frac{41\pi}{30} > \pi$

Then the bolded answers are all the critical numbers in the desired interval. Pick the answer that appears on that list.

and