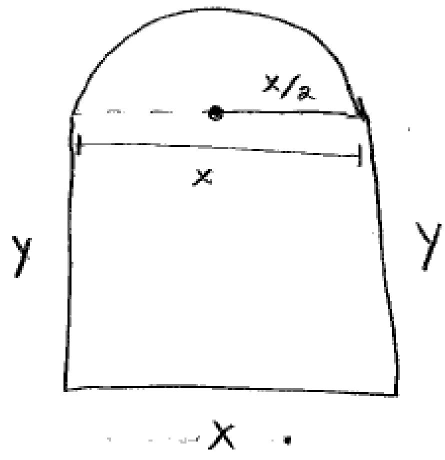
HW 25 #1



① Maximize area:

$$A = \underbrace{xy}_{\text{area of rectangle}} + \underbrace{\tfrac{1}{2}\left(\pi\left(\tfrac{x}{2}\right)^2\right)}_{\text{area of semicircle}}$$

② Constraint: Perimeter is 32 ft.

$$32 = \underbrace{x + 2y}_{\text{sides of rectangle}} + \underbrace{\tfrac{1}{2}\left(\cancel{2}\pi\left(\tfrac{x}{\cancel{2}}\right)\right)}_{\text{perimeter of semicircle}}$$

③ Solve constraint for y:

$$32 = x + 2y + \frac{\pi}{2}x$$
$$2y = 32 - x - \frac{\pi}{2}x$$
$$2y = 32 - x\left(1 + \frac{\pi}{2}\right)$$
$$\frac{1}{2}\cdot\left(2y = 32 - x\left(\frac{2+\pi}{2}\right)\right)\cdot\frac{1}{2}$$
$$\boxed{y = 16 - x\left(\frac{2+\pi}{4}\right)}$$

Plug into area:
$$A = x\left(16 - x\left(\frac{2+\pi}{4}\right)\right) + \frac{\pi}{8}x^2$$
$$A = 16x - x^2\left(\frac{2+\pi}{4}\right) + \frac{\pi}{8}x^2$$

④ Take derivative:
$$A' = 16 - \overset{1}{\cancel{2}}x\left(\frac{2+\pi}{\underset{2}{\cancel{4}}}\right) + \frac{\pi}{\underset{4}{\cancel{8}}}\left(\overset{1}{\cancel{2}}x\right)$$

⑤ Set $A' = 0$ and solve:

$$0 = 16 - x\left(\frac{2+\pi}{2}\right) + \frac{\pi}{4}x$$
$$0 = 16 + \frac{\pi}{4}x - \left(\frac{2+\pi}{2}\right)x$$
$$0 = 16 + x\left(\frac{\pi}{4} - \frac{2+\pi}{2}\right)$$
$$0 = 16 + x\left(\frac{\pi}{4} - \frac{2(2+\pi)}{4}\right)$$
$$0 = 16 + x\left(\frac{\pi - (4+2\pi)}{4}\right)$$
$$0 = 16 + x\left(\frac{\pi - 4 - 2\pi}{4}\right)$$
$$0 = 16 + x\left(\frac{-4-\pi}{4}\right)$$
$$0 = 16 + x\left(\frac{-(4+\pi)}{4}\right)$$

$$0 = 16 - x\left(\frac{4+\pi}{4}\right)$$
$$x\left(\frac{4+\pi}{4}\right) = 16$$
$$x = \frac{16}{\left(\frac{4+\pi}{4}\right)} = 16\cdot\frac{4}{4+\pi} = \frac{64}{4+\pi}$$

⑥ Find y:

$$y = 16 - \left(\frac{\overset{16}{\cancel{64}}}{4+\pi}\right)\left(\frac{2+\pi}{\underset{1}{\cancel{4}}}\right)$$
$$y = 16 - \frac{16(2+\pi)}{4+\pi}$$
$$y = \frac{16(4+\pi) - 16(2+\pi)}{4+\pi}$$
$$y = \frac{64 + \cancel{16\pi} - 32 - \cancel{16\pi}}{4+\pi}$$
$$y = \frac{32}{4+\pi}$$

Answer:
$$\boxed{x = \frac{64}{4+\pi} \qquad y = \frac{32}{4+\pi}}$$