Lesson 25: Optimization (Part 2)

Ex. 1 Mr. Woolsey is adding a window to the Atlantis conference room. He wants to construct it by adjoining an equilateral triangular window to the top of a rectangular window with width x and height y. If the perimeter of the window is 32 ft, find the dimensions that will allow the window to admit the most light.

Recall that the area of an equilateral triangle with side a is $A = \frac{\sqrt{3}}{4}\, a^2$.

Draw a picture!

Step 1: Determine the quantity to optimize.

Which quantity are we optimizing? (Circle one)

Area **Perimeter**

How are we optimizing the quantity?

Minimizing **Maximizing**

Write an equation for the quantity:

Step 2: Identify the constraint(s).

Which quantity is our constraint? **Area** **Perimeter**

Write an equation for the constraint:

Step 3a: Solve the constraint for one of the variables.

Step 3b: Substitute the expression from Step 3a into the equation in Step 1.

Step 4: Take the derivative.

Step 5: Set the derivative equal to 0 and solve for the variable to find the critical numbers.
(Remember that once we have the critical numbers, we can use the First or Second Derivative Test to identify which critical number corresponds to the relative min or max.)

Step 6: Compute the desired quantity.
What does the problem ask us to find? ________________________________

Ex. 2 Sam and Jack have a cylindrical Jello mold with a surface area of 25. What is the maximum volume that it can have?
Recall: the surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$ and the volume of a cylinder is $V = \pi r^2 h$.

Step 1: Determine the quantity to optimize.
Which quantity are we optimizing? (Circle one)

Volume **Surface Area**

How are we optimizing the quantity?

Minimizing **Maximizing**

Write an equation for the quantity:

Step 2: Identify the constraint(s).
Which quantity is our constraint? **Volume** **Surface Area**
Write an equation for the constraint:

Step 3a: Solve the constraint for one of the variables.

Step 3b: Substitute the expression from Step 3a into the equation in Step 1.

Step 4: Take the derivative.

Step 5: Set the derivative equal to 0 and solve for the variable to find the critical numbers.

Step 6: Compute the desired quantity.
What does the problem ask us to find? ____________________________________

<u>Ex. 3</u> A box with a square base and no top is to be built with a volume of 4 in^3. Find the dimensions of the box that require the least amount of material. How much material is required at the minimum?

Draw a picture!

Step 1: Determine the quantity to optimize.
Which quantity are we optimizing?

How are we optimizing the quantity?

Write an equation for the quantity:

Step 2: Identify the constraint(s).
Which quantity is our constraint?

Write an equation for the constraint:

Step 3a: Solve the constraint for one of the variables.

Step 3b: Substitute the expression from Step 3a into the equation in Step 1.

Step 4: Take the derivative.

Step 5: Set the derivative equal to 0 and solve for the variable to find the critical numbers.

Step 6: Compute the desired quantity.
What does the problem ask us to find? ______________________________________

Ex. 4 A box has a circular base. If the sum of the height of the box and the perimeter of the circular base is $27\,\pi$ in, what is the maximum possible volume?

Draw a picture!

Step 1: Determine the quantity to optimize.
Which quantity are we optimizing?

How are we optimizing the quantity?

Write an equation for the quantity:

Step 2: Identify the constraint(s).
Which quantity is our constraint?

Write an equation for the constraint:

Step 3a: Solve the constraint for one of the variables.

Step 3b: Substitute the expression from Step 3a into the equation in Step 1.

Step 4: Take the derivative.

Step 5: Set the derivative equal to 0 and solve for the variable to find the critical numbers.

Step 6: Compute the desired quantity.

What does the problem ask us to find? ___________________________________

Ex. 5 The SGC needs to make a cylindrical can that can hold precisely 4.1 L of liquid naquadah. If the entire can is to be made out of trinium, find the dimensions of the can that will minimize the cost.
Recall: the surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$ and the volume of a cylinder is $V = \pi r^2 h$.

Step 1: Determine the quantity to optimize.
Which quantity are we optimizing?

How are we optimizing the quantity?

Write an equation for the quantity:

Step 2: Identify the constraint(s).
Which quantity is our constraint?

Write an equation for the constraint:

Step 3a: Solve the constraint for one of the variables.

Step 3b: Substitute the expression from Step 3a into the equation in Step 1.

Step 4: Take the derivative.

Step 5: Set the derivative equal to 0 and solve for the variable to find the critical numbers.

Step 6: Compute the desired quantity.
What does the problem ask us to find? ____________________________________