Lesson 3: Finding Limits Graphically

Ex.1 Fill in the table and draw a graph to find

\[
\lim_{{x \to 2^-}} f(x) \\
\lim_{{x \to 2^+}} f(x)
\]

and

\[
\lim_{{x \to 2}} f(x)
\]

for \( f(x) = \begin{cases} 
  x^2 + 1 & x \leq 2 \\
  -2x + 1 & x > 2 
\end{cases} \).

Table:

<table>
<thead>
<tr>
<th>x</th>
<th>1.9</th>
<th>1.99</th>
<th>1.999</th>
<th>1.9999</th>
<th>2</th>
<th>2.0001</th>
<th>2.001</th>
<th>2.01</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>4.61</td>
<td>4.9601</td>
<td>4.9960</td>
<td>4.9996</td>
<td>-</td>
<td>-3.002</td>
<td>-3.02</td>
<td>-3.02</td>
<td>-3.2</td>
</tr>
</tbody>
</table>

To find \( f(1.9), f(1.99), f(1.999), f(1.999) \), we need to plug 1.9, 1.99, 1.999, 1.999 into \( x^2 + 1 \) because 1.9, 1.99, 1.999, 1.999 are all less than 2.

To find \( f(2.0001), f(2.001), f(2.01), f(2.1) \), we need to plug 2.0001, 2.001, 2.01, 2.1 into \(-2x + 1\) because 2.0001, 2.001, 2.01, 2.1 are all greater than 2.

Looking at the values on the left side of the table, we see that \( \lim_{{x \to 2^-}} f(x) = 5 \), and looking at the values on the right side of the table, we see that \( \lim_{{x \to 2^+}} f(x) = -3 \).

Since the left and right sided limits are not equal, \( \lim_{{x \to 2}} f(x) \) DNE.

Graph:

\( x^2 + 1 \) is a parabola with vertex (0,1)

\(-2x + 1\) is a line with slope -2 and \( y \)-intercept (0,1).
Ex.2 Use the graph to find the following:

- \( \lim_{x \to -2^-} f(x) = 0 \)
- \( \lim_{x \to -2^+} f(x) = 0 \)
- \( \lim_{x \to -2} f(x) = 0 \)
- \( f(-2) = 0 \)

- \( \lim_{x \to 0^-} f(x) = 2 \)
- \( \lim_{x \to 0^+} f(x) = 0 \)
- \( \lim_{x \to 0} f(x) = \text{DNE} \)
- \( f(0) = 2 \)

- \( \lim_{x \to 2^-} f(x) = 4 \)
- \( \lim_{x \to 2^+} f(x) = 4 \)
- \( \lim_{x \to 2} f(x) = 4 \)
- \( f(2) = \text{UNDEFINED} \)
Ex. 3 Use the graph to find the following:

- \( \lim_{x \to -1^-} f(x) = 1 \)
- \( \lim_{x \to -1^+} f(x) = 1 \)
- \( \lim_{x \to -1} f(x) = 1 \)
- \( f(-1) = -1 \)
- \( \lim_{x \to 0^-} f(x) = \infty \)
- \( \lim_{x \to 0^+} f(x) = -2 \)
- \( \lim_{x \to 0} f(x) = \text{DNE} \)
- \( f(0) = -2 \)
- \( \lim_{x \to 2^-} f(x) = 0 \)
- \( \lim_{x \to 2^+} f(x) = 0 \)
- \( \lim_{x \to 2} f(x) = 0 \)
- \( f(2) = 0 \)

Ex. 4 Graph \( f(x) = \begin{cases} \cos x & x < 0 \\ \sin x & 0 \leq x < \pi \\ \cos x & \pi < x \end{cases} \) to find the following:
Ex. 5  Graph \( f(x) = \frac{1}{x-1} \) to find the following:

\[
\begin{align*}
\lim_{x \to 1^-} f(x) &= -\infty \\
\lim_{x \to 1^+} f(x) &= \infty \\
\lim_{x \to 1} f(x) &= \text{DNE} \\
\end{align*}
\]

\( f(1) = \text{UNDEFINED} \)

Ex. 6  Graph \( f(x) = \frac{1}{(x-1)^2} \) to find the following:

\[
\begin{align*}
\lim_{x \to 1^-} f(x) &= \infty \\
\lim_{x \to 1^+} f(x) &= \infty \\
\lim_{x \to 1} f(x) &= \infty \\
\end{align*}
\]

\( f(1) = \text{UNDEFINED} \)

Notice the difference between examples 5 and 6. The squared in the denominator for example 6 makes it so that all values of the function will be positive. Recall learning about the multiplicity of functions in algebra or precalculus.

Ex. 7  Graph \( f(x) = |x|, \ x \neq 0 \) to find the following:
Ex. 8 Graph $f(x) = \begin{cases} \frac{1}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$ to find the following:

- $\lim_{x \to 0^-} f(x) = \infty$
- $\lim_{x \to 0^+} f(x) = \infty$
- $\lim_{x \to 0} f(x) = \infty$
- $f(0) = 0$