Lesson 3: Finding Limits Graphically

Ex.1 Fill in the table and draw a graph to find

$$\lim_{x\to 2^-} f(x)$$

$$\lim_{x\to 2^+} f(x)$$

and

$$\lim_{x\to 2} f(x)$$

for $f(x) = \begin{cases} x^2+1 & x \leq 2 \\ -2x+1 & x > 2 \end{cases}.$

Table:

x	**1.9**	**1.99**	**1.999**	**1.9999**	**2**	**2.0001**	**2.001**	**2.01**	**2.1**
f(x)	4.61	4.9601	4.9960	4.9996	**-**	-3.0002	-3.002	-3.02	-3.2

To find $f(1.9), f(1.99), f(1.999), f(1.999)$, we need to plug $1.9, 1.99, 1.999, 1.999$ into x^2+1 because $1.9, 1.99, 1.999, 1.999$ are all less than 2.

To find $f(2.0001), f(2.001), f(2.01), f(2.1)$, we need to plug $2.0001, 2.001, 2.01, 2.1$ into $-2x+1$ because $2.0001, 2.001, 2.01, 2.1$ are all greater than 2.

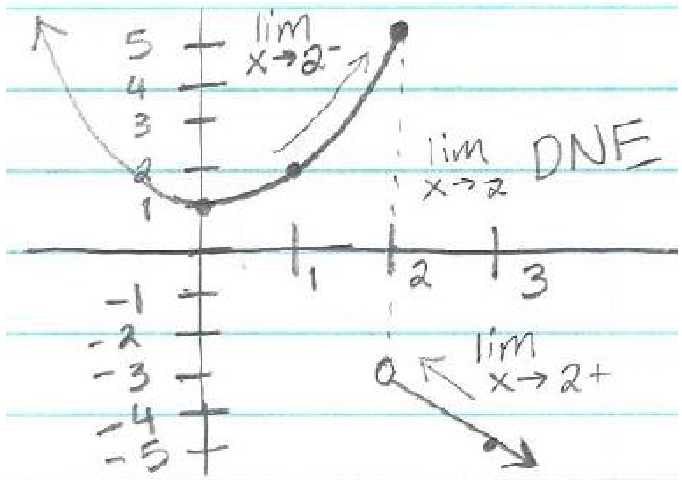
Looking at the values on the left side of the table, we see that $\lim_{x\to 2^-} f(x) = 5$, and looking at the values on the right side of the table, we see that $\lim_{x\to 2^+} f(x) = -3$.

Since the left and right sided limits are not equal, $\lim_{x\to 2} f(x)$ DNE.

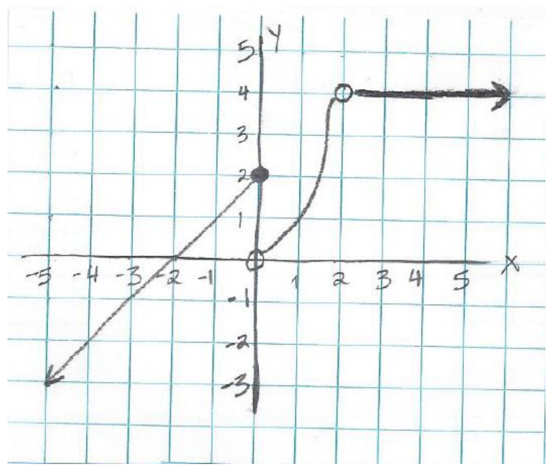
Graph:

x^2+1 is a parabola with vertex (0,1)

$-2x+1$ is a line with slope -2 and y-intercept (0,1).



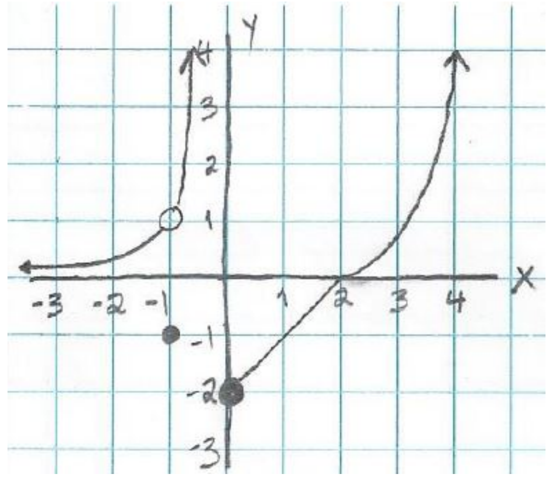
Ex.2 Use the graph to find the following:



- $\lim_{x\to -2^-} f(x) = 0$
- $\lim_{x\to -2^+} f(x) = 0$
- $\lim_{x\to -2} f(x) = 0$
- $f(-2) = 0$

- $\lim_{x\to 0^-} f(x) = 2$
- $\lim_{x\to 0^+} f(x) = 0$
- $\lim_{x\to 0} f(x) =$DNE
- $f(0) = 2$
- $\lim_{x\to 2^-} f(x) = 4$
- $\lim_{x\to 2^+} f(x) = 4$
- $\lim_{x\to 2} f(x) = 4$
- $f(2) =$UNDEFINED

Ex.3 Use the graph to find the following:

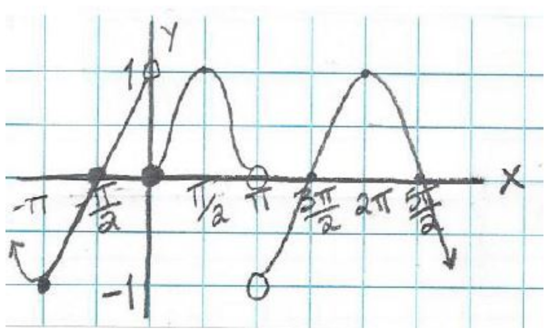


- $\lim_{x\to -1^-} f(x) = 1$
- $\lim_{x\to -1^+} f(x) = 1$
- $\lim_{x\to -1} f(x) = 1$
- $f(-1) = -1$

- $\lim_{x\to 0^-} f(x) = \infty$
- $\lim_{x\to 0^+} f(x) = -2$
- $\lim_{x\to 0} f(x) =$DNE
- $f(0) = -2$

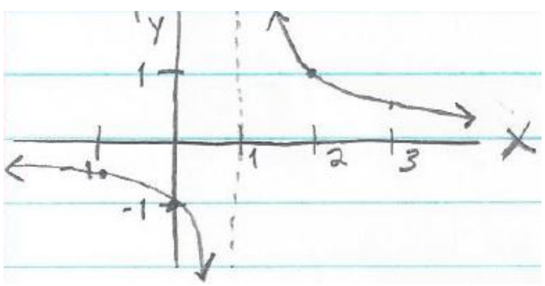
- $\lim_{x\to 2^-} f(x) = 0$
- $\lim_{x\to 2^+} f(x) = 0$
- $\lim_{x\to 2} f(x) = 0$
- $f(2) = 0$

Ex.4 Graph $f(x) = \begin{cases} \cos x & x < 0 \\ \sin x & 0 \le x < \pi \\ \cos x & \pi < x \end{cases}$ to find the following:



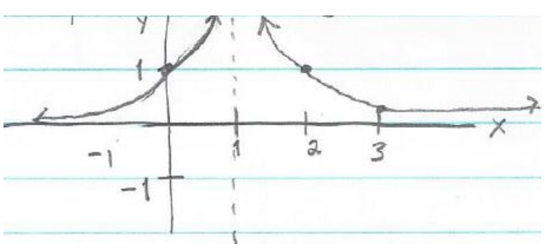
- $\lim_{x\to 0^-} f(x) = 1$
- $\lim_{x\to 0^+} f(x) = 0$
- $\lim_{x\to 0} f(x) =$DNE
- $f(0) = 0$
- $\lim_{x\to \pi^-} f(x) = 0$
- $\lim_{x\to \pi^+} f(x) = -1$
- $\lim_{x\to \pi} f(x) =$DNE
- $f(\pi) =$ UNDEFINED

<u>Ex.5</u> Graph $f(x) = \frac{1}{x-1}$ to find the following:



- $\lim_{x\to 1^-} f(x) = -\infty$
- $\lim_{x\to 1^+} f(x) = \infty$
- $\lim_{x\to 1} f(x) =$DNE
- $f(1) =$ UNDEFINED

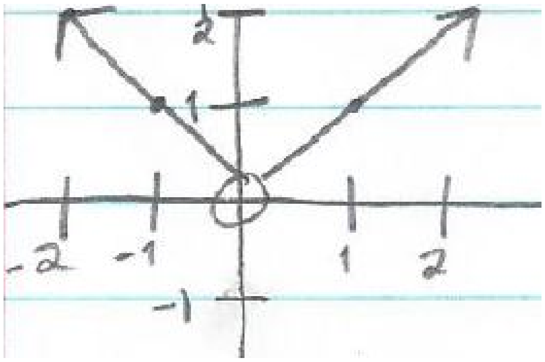
<u>Ex.6</u> Graph $f(x) = \frac{1}{(x-1)^2}$ to find the following:



- $\lim_{x\to 1^-} f(x) = \infty$
- $\lim_{x\to 1^+} f(x) = \infty$
- $\lim_{x\to 1} f(x) = \infty$
- $f(1) =$ UNDEFINED

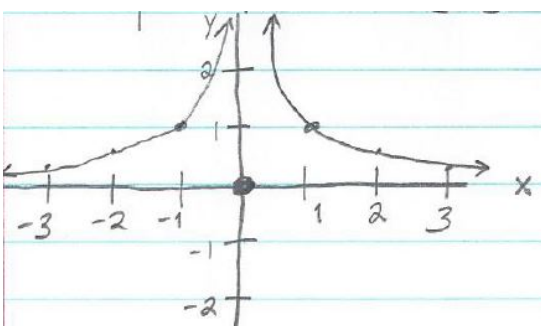
Notice the difference between examples 5 and 6. The squared in the denominator for example 6 makes it so that all values of the function will be positive. Recall learning about the multiplicity of functions in algebra or precalculus.

<u>Ex.7</u> Graph $f(x) = |x|, x \neq 0$ to find the following:



- $\lim_{x \to 0^-} f(x) = 0$
- $\lim_{x \to 0^+} f(x) = 0$
- $\lim_{x \to 0} f(x) = 0$
- $f(0) =$ UNDEFINED

<u>Ex.8</u> Graph $f(x) = \begin{cases} \frac{1}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$ to find the following:



- $\lim_{x \to 0^-} f(x) = \infty$
- $\lim_{x \to 0^+} f(x) = \infty$
- $\lim_{x \to 0} f(x) = \infty$
- $f(0) = 0$