MA 266 Lecture 1

Section 1.1 Mathematical Models; Direction Fields

Question: What is a differential equation?

A **differential equation** is

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Example 1. ***(Types of equations)***

1. *Find* x *in* $\quad x^2 + 2x + 1 = 0.$

2. *Find* $f(t)$ *in* $\quad f(t)\cos(t) = e^t - \sin(t).$

3. *Find* $y(t)$ *in* $\quad y'' + 3y' = e^t.$

Question: Why do we study differential equations?

- Many principles or laws in physics are relations involving _____.
- In mathematical terms, relations are _____, and rates are _____. Equations containing derivatives are _____.
- A differential equation that describes certain physical process is often called a _____.

Example 2. ***(An example of mathematical model — A falling object)***

Consider an object with a mass m falling near the sea level. Formulate a differential equation to model its motion.

- **Notations**

- **Physical Law: Newton's second law**

- **Forces that acted on the object**

Remark The falling object model contains three constants: m, g, and γ

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Direction Fields

We let $m = 10kg$ and $\gamma = 2kg/s$ in the falling object model, so it becomes

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}.$$

Basic idea of direction fields:

How to construct Direction Fields?

If we let $v = 40$, then

If we let $v = 50$, then

Note that if $9.8 - \frac{v}{5} = 0$, then

Remarks on Direction Fields

Direction fields are valuable tools in studying differential equations of the form

Two things about direction fields

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A MATLAB Implementation on Direction Fields

1. Download the MATLAB file **dfield8.m** from

   `http://math.rice.edu/~dfield`

2. Type **dfield8**, at MATLAB command window.
3. In the popup window, enter your differential equations, and the range of independent and dependent variables.
4. Hit **Proceed** to see the direction field of your differential equation.

Example 3. *Draw a direction field of the each of the following differential equations, then determine the behavior of the solution as* $t \to \infty$.

$$\textit{(1)}\ y' = 3 - 2y, \qquad \textit{(2)}\ y' = 3 + 2y, \qquad \textit{(3)}\ y' = -y(5 - y).$$