MA 266 Lecture 6

Section 2.3 Modeling with First Order Equations

In this section, we consider several mathematical models using first order differential equations.

Steps in the process of mathematical modeling:

1. **Construction of the Model**

2. **Analysis of the Model**

3. **Comparison with Experiment or Observation**

Example 1. ***(Mixing)***

At time $t = 0$, a tank contains Q_0 lb of salt dissolved in 100 gallons of water. Assume that water containing 1/4 lb of salt/gal is entering the tank at a rate of r gal/min and that the well-stirred mixture is draining from the tank at the same rate. *(continue next page)*

An illustration of water tank in Example 1

Answer the following questions.

1. Set up the initial value problem that describes this flow process.

2. Find the amount of salt $Q(t)$ in the tank at any time and find the limiting amount Q_L that is present after a very long time.

(Example 1 continues next page)

3. If $r = 3$, $Q_0 = 2Q_L$, find the time T after which the salt level is within 2% of Q_L.

4. Also find the flow rate that is required if the value of T is not to exceed 45 min.

Remark

- Applications of this model include

- In a more general model,

Example 2. ***(An Extension of Example 1: Problem 4)***

A tank with a capacity of 500 gal originally contains 200 gal of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow.