MA 266 Lecture 9

Section 2.5 Autonomous Equations and Population Dynamics

A differential equation is called _____ if it has the form

We know how to solve autonomous equations because they are _____. In this section, we will use geometrical methods to obtain important qualitative information about differential equations of this type.

1. Population Dynamics: Exponential Growth

Let $y = \phi(t)$ be the population of a given species at time t.

Remark

- This model suggests that

- The ideal conditions cannot continue indefinitely,

2. Population Dynamics: Logistic Growth

This model takes into account the fact that the growth rate depends on _____.

Replace the constant r by a function

We want the the function $h(y)$ to be close to r when y is small.

The above equation is called _____. It can be written as

To find constant solution of the equation, we let

The solutions are called _____ of the differential equation, or _____ of the function $f(y)$.

3. Plot of $f(y)$ vs y, phase line, plot of solutions

- If $0 < y < K$,
- If $y > K$

plot of $f(y)$ vs y phase line plot of solutions

Remark No solution will

Next, let us analytically solve the initial value problem

$$\frac{dy}{dt} = r(1 - \frac{y}{K})y, \qquad y(0) = y_0.$$

For each $y_0 > 0$, the solution approaches the _____. Hence, we say that

- the constant solution $y = K$ is an _____.
- the constant solution $y = 0$ is an _____.

Example 1. *(Problem #5) Consider the differential equation*

$$\frac{dy}{dt} = e^{-y} - 1$$

Sketch the graph of $f(y)$ versus y, determine the critical points, and classify each one as asymptotically stable or unstable. Draw the phase line, and sketch several graphs of solutions.