MA 266 Lecture 11

Section 2.7 Numerical Approximation: Euler's Method

In this section, we introduce a numerical method for solving the first order initial value problem

$$\frac{dy}{dt} = f(t, y), \qquad y(t_0) = y_0.$$

The method is called ________________________ or ________________________.

How to use tangent lines to approximate the solution $y = \phi(t)$?

- Start with the initial point (t_0, y_0),

- We want to continue this process with the point $(t_1, \phi(t_1))$, however,

- The general expression for the tangent line starting at (t_n, y_n) is

  The approximate value y_{n+1} at t_{n+1} in terms of t_n and y_n is

  If we denote $f_n =$

  If step size between the point t_0, t_1, t_2, $\cdots$ is uniform,

Remark. Euler's method will generate a sequence of values y_1, y_2, $\cdots$,

Example 1. *Consider the initial value problem*

$$\frac{dy}{dt} = 3 - 2t - 0.5y, \qquad y(0) = 1.$$

Use Euler's method with step size $h = 0.2$ *to find approximate values of solution at* $t = 0.2, 0.4, 0.6, 0.8,$ *and* 1*. Compare them with the corresponding values of the actual solution of the IVP.*