MA 266 Lecture 13

Section 3.1 Homogeneous Equations with Constant Coefficients

Terminologies

Starting from this section, we study the second order differential equation of the form

The above equation is called \underline{linear} if

For a linear equation, we usually write it as

- (standard form)
- (general form)

An initial value problem consists of a differential equation and *a pair of* initial conditions:

A second order linear equation is called homogeneous if it has the form

Otherwise, it is called _____.

In this section, we focus on in homogeneous equations with constant coefficients, i.e.,

Example 1. Solve the initial value problem

$$\begin{cases} y'' - y = 0\\ y(0) = 2, \quad y'(0) = -1. \end{cases}$$

In general, consider the homogeneous equation

$$ay'' + by' + cy = 0,$$

where a, b, c are given constants.

The equation $ar^2 + br + c = 0$ is called ______. If the characteristic equation has two *distinct real* roots r_1 and r_2 , then

Example 2. Solve the initial value problem

$$\begin{cases} y'' + 5y' + 6Y = 0\\ y(0) = 2, \quad y'(0) = 3. \end{cases}$$

Example 3. Consider the second order equation

$$y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0.$$

Determine the value of α , if any, for which all solutions tend to zero as $t \to \infty$. Also determine the value of α . if any, for which all (nonzero) solutions become unbounded as $t \to \infty$.