MA 266 Lecture 15

Section 3.3 Complex Roots of Characteristic Equation

Review. Consider the linear homogeneous equation with constant coefficient

$$ay'' + by' + cy = 0.$$

The characteristic equation is

If the discriminant $b^2 - 4ac > 0$, then

In this section we study the case when the discriminant $b^2 - 4ac < 0$.

Question: What does it mean to raise the number e to a complex power?

Euler's formula:

Euler's formula can be derived as follows

Remarks. The following identities are immediate results from Euler's formula

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Example 1. Find the general solution of

$$y'' + y' + 9.25y = 0.$$

Then find the solution satisfies the initial conditions

$$y(0) = 2, \qquad y'(0) = 8.$$

The general case (complex roots)

If the roots of characteristic equation is $\lambda \pm \mu$,

Example 2. Solve the initial value problem

$$y'' + 4y' + 5y = 0,$$
 $y(0) = 1,$ $y'(0) = 0.$