MA 266 Lecture 16

Section 3.4 Repeated Roots; Reduction of Order

Review. Consider the linear homogeneous equation with constant coefficient

$$ay'' + by' + cy = 0.$$

The characteristic equation is

If the discriminant $b^2 - 4ac > 0$, then

If the discriminant $b^2 - 4ac < 0$, then

In this section we study the third case when the discriminant $b^2 - 4ac = 0$.

Question: How to find another solution?

We consider a specific case in the next example.

Example 1. *Solve the differential equation*

$$y'' + 4y' + 4y = 0.$$

The general case (repeated roots)

If the characteristic equation have two repeated roots $r_1 = r_2 = -\frac{b}{2a}$,

Example 2. *Solve the initial value problem*

$$y'' - y' + 0.25y = 0, \qquad y(0) = 2, \qquad y'(0) = \frac{1}{3}.$$

Summary For second order linear homogeneous equations with constant coefficients

$$ay'' + by' + cy = 0,$$

Let r_1 and r_2 be roots of characteristic equation $ar^2 + br + c = 0$.

- If r_1 and r_2 are real, and $r_1 \neq r_2$, then

- If r_1 and r_2 are real, and $r_1 = r_2$, then

- If r_1 and r_2 are complex conjugate $\lambda \pm i\mu$, then

Reduction of Order

Suppose we know $y_1(t)$ is a solution of the linear homogeneous equation (not necessarily constant coefficients):

$$y'' + p(t)y' + q(t)y = 0.$$

To find another solution, we let

Example 3. *Given that* $y_1 = t$ *is a solution of*

$$t^2 y'' + 2ty' - 2y = 0.$$

find a fundamental set of solutions.