# MA 266 Lecture 17

## Section 3.5 Nonhomogeneous Equations; Method of Undetermined Coefficients

We consider the nonhomogeneous equation

$$L[y] = y'' + p(t)y' + q(t)y = g(t).$$

The corresponding homogeneous equation is

The structure of the solution is summarized in the following theorem

**Theorem** If $Y_1$ and $Y_2$ are two solutions of nonhomogeneous equation

To see this,

**Theorem** The general solution of nonhomogeneous equation can be written as

**Remark.** To solve a nonhomogeneous equation, we do three things:

1. Find

2. Find

3. Form

**Method of Undetermined Coefficients**

There are generally two methods to find a particular solution $Y(t)$:

1. _____ and 2. _____.

There are two steps for Method of undetermined coefficients

- Make an initial assumption

- Substitute the assumed expression into the equation.

**1. $g(t)$ is an exponential function**

**Example 1.** *Find a particular solution of*

$$y'' - 3y' - 4y = 3e^{2t}.$$

**2.** $g(t)$ **is a sine or cosine function**

**Example 2.** *Find a particular solution of*

$$y'' - 3y' - 4y = 2\sin(t).$$

**3.** $g(t)$ **is the product of exponential and trig functions**

**Example 3.** *Find a particular solution of*

$$y'' - 3y' - 4y = -8e^t \cos(2t).$$

**4. $g(t)$ is sum of two terms**

If the right hand side $g(t)$ is the sum of two terms, $g(t) = g_1(t) + g_2(t)$

**Example 4.** *Find a particular solution of*

$$y'' - 3y' - 4y = 3e^{2t} + 2\sin(t) - 8e^t \cos(2t).$$

**5. One difficulty**

**Example 5.** *Find a particular solution of*

$$y'' - 3y' - 4y = 2e^{-t}.$$

**Summary**. The particular solution of $ay'' + by' + cy = g(t)$

- If $g(t) = P_n(t) = a_0 t^n + a_1 t^{n-1} + \cdots + a_n$, then

- If $g(t) = P_n(t)e^{\alpha t}$, then

- If $g(t) = P_n(t)e^{\alpha t}\sin(\beta t)$ or $g(t) = P_n(t)e^{\alpha t}\cos(\beta t)$, then

Here the number $s$ is