Section 3.5 Nonhomogeneous Equations; Method of Undetermined Coefficients

We consider the nonhomogeneous equation

\[ L[y] = y'' + p(t)y' + q(t)y = g(t). \]

The corresponding homogeneous equation is

The structure of the solution is summarized in the following theorem

**Theorem** If \( Y_1 \) and \( Y_2 \) are two solutions of nonhomogeneous equation

To see this,

**Theorem** The general solution of nonhomogeneous equation can be written as

**Remark.** To solve a nonhomogeneous equation, we do three things:

1. Find

2. Find

3. Form
Method of Undetermined Coefficients

There are generally two methods to find a particular solution \( Y(t) \):

1. \( \) and 2. \( \).

There are two steps for Method of undetermined coefficients

• Make an initial assumption

• Substitute the assumed expression into the equation.

1. \( g(t) \) is an exponential function

Example 1. Find a particular solution of

\[
y'' - 3y' - 4y = 3e^{2t}.
\]
2. \( g(t) \) is a sine or cosine function

**Example 2.** Find a particular solution of

\[
y'' - 3y' - 4y = 2\sin(t).
\]
3. $g(t)$ is the product of exponential and trig functions

**Example 3.** Find a particular solution of

$$y'' - 3y' - 4y = -8e^t \cos(2t).$$
4. $g(t)$ is sum of two terms

If the right hand side $g(t)$ is the sum of two terms, $g(t) = g_1(t) + g_2(t)$

Example 4. Find a particular solution of

$$y'' - 3y' - 4y = 3e^{2t} + 2\sin(t) - 8e^t \cos(2t).$$
5. One difficulty

**Example 5.** Find a particular solution of

\[ y'' - 3y' - 4y = 2e^{-t}. \]

**Summary.** The particular solution of \( ay'' + by' + cy = g(t) \)

- If \( g(t) = P_n(t) = a_0 t^n + a_1 t^{n-1} + \cdots + a_n \), then
- If \( g(t) = P_n(t)e^{\alpha t} \), then
- If \( g(t) = P_n(t)e^{\alpha t} \sin(\beta t) \) or \( g(t) = P_n(t)e^{\alpha t} \cos(\beta t) \), then

Here the number \( s \) is