Section 3.8  Forced Vibration

In this section, we consider the situation in which a periodic external force is applied to a spring-mass system.

Forced Vibration with Damping

Example 1. Suppose that the motion of a certain spring-mass system satisfies the differential equation:

\[ u'' + u' + 1.25u = 3 \cos(t), \quad u(0) = 2, \quad u'(0) = 3. \]

Find the solution of this initial value problem and describe the behavior of the solution for large \( t \).
In general, the equation of motion of a spring-mass system subject to an external force $F(t)$ is

Forced Vibration without Damping
In this case, the motion is governed by the equation

$$mu'' + ku = F_0 \cos(\omega t).$$

- If $\omega \neq \omega_0$,

- If $\omega = \omega_0$, 

Example 2. *(Problem 10)* A mass weighing 8 lb stretches a spring 6 in. The mass is acted on by an external force of $8 \sin(8t)$ lb. If the mass is pulled down 3 in and then released, determine the position of the mass at any time. Determine the first four times at which the velocity of the mass is zero.