MA 266 Lecture 22

4.1 General Theory of nth Order Linear Equations

An nth order linear differential equation is

If P_0 is nowhere zero in the interval I, the equation can be written as

The initial conditions are

Theorem (Existence and Uniqueness) If p_1, p_2, $\cdots$, p_n and g are

The n-th order linear homogeneous equation is

If y_1, y_2, $\cdots$, y_n are solutions, then

The Wronskian of solutions y_1, y_2, $\cdots$, y_n are

Theorem If the functions $y_1,\ y_2,\ \cdots,\ y_n$ are solutions of the homogeneous equation and if

Remark A set of solutions $y_1,\ y_2,\ \cdots,\ y_n$ whose Wronskian is nonzero are called a

Linear dependency of functions

The functions $f_1,\ f_2,\ \cdots,\ f_n$ are said to be ______________________ on an interval I, if

Example 1. *Determine whether the functions $f_1(t) = 1,\ f_2(t) = t,$ and $f_3 = t^2$ are linearly independent or dependent on the interval $I : -\infty < t < \infty$.*

Example 2. *Determine whether the functions $f_1(t) = 1,\ f_2(t) = 2 + t,\ f_3 = 3 - t^2,$ and $f_4 = 4t + t^2$ are linearly independent or dependent on the interval $I : -\infty < t < \infty$.*

Theorem

- If the functions y_1, y_2, $\cdots$, y_n form a fundamental set of solutions

- If y_1, y_2, $\cdots$, y_n are linearly independent on some interval I

For the nonhomogeneous equation

$$\frac{d^n y}{dt^n} + p_1(t)\frac{d^{n-1}y}{dt^{n-1}} + \cdots + p_{n-1}(t)\frac{dy}{dt} + p_n(t)y = g(t),$$

the general solution is

4.2 Homogeneous Equations with Constant Coefficients

We consider the nth order linear homogeneous equation with constant coefficients:

if $y = e^{rt}$ is a solution, then

Real and distinct roots

If the characteristic equation has n real and distinct roots, then

Example 3. *Find the general solution of*

$$2y''' - 4y'' - 2y' + 4y = 0.$$

Complex roots

If the characteristic equation has complex roots, then

Example 4. *Find the general solution of*

$$y^{(4)} - y = 0.$$

Repeated roots

If the characteristic equation has a repeated root r_1 and multiplicity is s, then

If the characteristic equation has a pair of complex roots $\lambda + i\mu$ repeated s times, then

Example 5. *Find the general solution of*

$$y^{(4)} + 2y'' + y = 0.$$