MA 266 Lecture 31

7.3 Linear Dependence, Eigenvalues, Eigenvectors

Linear Dependence

A set of k vectors $\mathbf{x}^{(1)}, \cdots,$ and $\mathbf{x}^{(k)}$ is said to be ________________________ if

On the other hand, if the only set $c_1, \cdots, c_k$ satisfying the equation is

then $\mathbf{x}^{(1)}, \cdots,$ and $\mathbf{x}^{(k)}$ is said to be ________________________.

For vector functions $\mathbf{x}^{(1)}(t), \cdots,$ and $\mathbf{x}^{(k)}(t)$, they are said to be **linearly dependent** on $\alpha < t < \beta$ if

Example 1. *Verify that the following vectors* $\mathbf{x}^{(1)}(t)$ *and* $\mathbf{x}^{(2)}(t)$ *are linearly independent on the interval* $0 < t < 1$

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} e^t \\ te^t \end{pmatrix}, \quad \mathbf{x}^{(2)}(t) = \begin{pmatrix} 1 \\ t \end{pmatrix}.$$

Eigenvalues and Eigenfunctions

The equation $A\mathbf{x} = \mathbf{y}$ can be viewed as a linear transform that maps a given vector $\mathbf{x}$ into a new vector $\mathbf{y}$. Vectors that are transformed into multiples of themselves are important in many applications. To find such vectors, we let $\mathbf{y} = \lambda\mathbf{x}$, then

Example 2. *Find the eigenvalues and eigenvectors of the matrix*

$$A = \begin{pmatrix} 3 & -1 \\ 4 & -2 \end{pmatrix}$$

7.4 Theory of System of First Order Linear Equations

The general form of a system of n first order linear equations is

$$\begin{aligned} x_1' &= \\ \vdots &= \vdots \\ x_n' &= \end{aligned}$$

We can write it in matrix form

The corresponding homogeneous system is

Principle of Superposition If the vector functions $\mathbf{x}^{(1)}(t), \cdots, \mathbf{x}^{(n)}(t)$ are solutions of the homogeneous system, then

The Wronskian of these n functions are

We say the vector functions $\mathbf{x}^{(1)}(t), \cdots, \mathbf{x}^{(n)}(t)$ are solutions form **a fundamental set of solutions** if

In this case, each solution $\mathbf{x}(t)$ of the homogeneous system can be express as

If $\mathbf{x}_p(t)$ is a particular solution of the nonhomogeneous system, the general solution is