# MA 266 Lecture 33

## 7.6 Complex Eigenvalues

In this section, we consider the system of linear homogeneous equations with constant coefficients. We focus on the case that the coefficient matrix has complex eigenvalues.

**Example 1.** *Find the general solution*

$$\mathbf{x}' = \begin{pmatrix} -\frac{1}{2} & 1 \\ -1 & -\frac{1}{2} \end{pmatrix} \mathbf{x}.$$

In general, we consider the system

$$\mathbf{x}' = A\mathbf{x}.$$

where $A$ has a pair of complex conjugate eigenvalues $r_{1,2} = \lambda \pm i\mu$.

**phase portrait**

If $A$ is a $2 \times 2$ matrix, we can visualize the solution is $x_1x_2$-plane, called **phase plane**, by evaluating $A\mathbf{x}$ at a large number of points. More precisely, we can include some solution curves, and a plot shows sample solution curves for a given system is called a **phase portrait**.

- If $r_1$, $r_2$ are real and have same sign

- If $r_1$, $r_2$ are real and have opposite sign

- If $r_{1,2} = \lambda \pm i\mu$

**Example 2.** *Consider the system*

$$\mathbf{x}' = \begin{pmatrix} \alpha & 2 \\ -2 & 0 \end{pmatrix} \mathbf{x}.$$

*Describe how the solution depends qualitatively on $\alpha$. In particular, find the critical values of $\alpha$ at which the qualitative behavior of the trajectory in the phase plane changes markedly.*