Section 1.1 Systems of Linear Equations

Definitions

• The equation
  \[ a_1x_1 + a_2x_2 + \cdots + a_nx_n = b \]
  is called a ________________.

  A sequence of numbers \(s_1, s_2, \cdots, s_n\) such that (1) is satisfied when \(x_1 = s_1, x_2 = s_2,\)
  \(\cdots, x_n = s_n\) is called ________________.

• More generally, the following system of equations
  \[
  \begin{array}{rcl}
  a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &=& b_1 \\
  a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &=& b_2 \\
  \vdots & & \vdots \\
  a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &=& b_m
  \end{array}
  \]
  is called a ________________.

  A ________________ to the linear system (2) is a sequence of \(n\) numbers \(s_1, s_2, \cdots, s_n\)
  which satisfies each equation in (2) when \(x_1 = s_1, x_2 = s_2, \cdots, x_n = s_n\).

• If the linear system (2) has no solution, it is said to be ________________.

  If the linear system (2) has a solution, it is called ________________.

• If \(b_1 = b_2 = \cdots = b_m = 0\), then (2) is called a ________________; otherwise
  it is called a ________________.

• Note that \(x_1 = x_2 = \cdots = x_n = 0\) is always a solution to a homogeneous system, and
  it is called the ________________.

  A nonzero solution to a homogeneous system is called a ________________.  

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• If there is another system of $r$ linear equations in $n$ unknowns:

\[
\begin{align*}
    c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n &= d_1 \\
    c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_n &= d_2 \\
    \vdots & \quad \vdots \\
    c_{r1}x_1 + c_{r2}x_2 + \cdots + c_{rn}x_n &= d_r
\end{align*}
\]

has exactly the same solution to (2), then we say they are ____________________.

**Method of Elimination**

*idea:* eliminating some variables by adding a multiple of one equation to another to make an equivalent system which is simpler to solve.

**Example 1.** Solve the linear system

\[
\begin{align*}
    x - 3y &= -7 \\
    2x - 6y &= 7
\end{align*}
\]

**Example 2.** Solve the linear system

\[
\begin{align*}
    x + 2y + 3z &= 6 \\
    2x - 3y + 2z &= 14 \\
    3x + y - z &= -2
\end{align*}
\]
Example 3. Solve the linear system

\[\begin{align*}
x + 2y - 3z &= -4 \\
2x + y - 3z &= 4
\end{align*}\]

Remark: a linear system may have

A geometrical explanation

Consider a linear system of two equations in two unknowns \(x\) and \(y\):

\[\begin{align*}
a_1x + b_1y &= c_1 \\
a_2x + b_2y &= c_2
\end{align*}\]