MA 265 Lecture 1

Section 1.1 Systems of Linear Equations

Definitions

- The equation
$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b \tag{1}$$
is called a ___________________.

  A sequence of numbers s_1, s_2, $\cdots$, s_n such that (1) is satisfied when $x_1 = s_1$, $x_2 = s_2$, $\cdots$, $x_n = s_n$ is called ___________________.

- More generally, the following system of equations
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\quad \vdots \qquad\qquad &= \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned} \tag{2}$$
is called a ____.

  A ___________________ to the linear system (2) is a sequence of n numbers s_1, s_2, $\cdots$, s_n which satisfies each equation in (2) when $x_1 = s_1$, $x_2 = s_2$, $\cdots$, $x_n = s_n$.

- If the linear system (2) has no solution, it is said to be ___________________.

  If the linear system (2) has a solution, it is called ___________________.

- If $b_1 = b_2 = \cdots = b_m = 0$, then (2) is called a _________________________; otherwise it is called a _________________________.

- Note that $x_1 = x_2 = \cdots = x_n = 0$ is always a solution to a homogeneous system, and it is called the ___________________.

  A nonzero solution to a homogeneous system is called a ___________________.

- If there is another system of r linear equations in n unknowns:

$$\begin{aligned} c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n &= d_1 \\ c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_n &= d_2 \\ \vdots \qquad\quad \vdots \qquad\quad &= \vdots \\ c_{r1}x_1 + c_{r2}x_2 + \cdots + c_{rn}x_n &= d_r \end{aligned} \tag{3}$$

  has exactly the same solution to (2), then we say they are __________________.

Method of Elimination

idea: eliminating some variables by adding a multiple of one equation to another to make an equivalent system which is simpler to solve.

Example 1. *Solve the linear system*

$$\begin{aligned} x - 3y &= -7 \\ 2x - 6y &= 7 \end{aligned}$$

Example 2. *Solve the linear system*

$$\begin{aligned} x + 2y + 3z &= 6 \\ 2x - 3y + 2z &= 14 \\ 3x + y - z &= -2 \end{aligned}$$

Example 3. *Solve the linear system*

$$\begin{aligned} x + 2y - 3z &= -4 \\ 2x + y - 3z &= 4 \end{aligned}$$

Remark: a linear system may have

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A geometrical explanation

Consider a linear system of two equations in two unknowns x and y:

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}$$