MA 265 Lecture 12

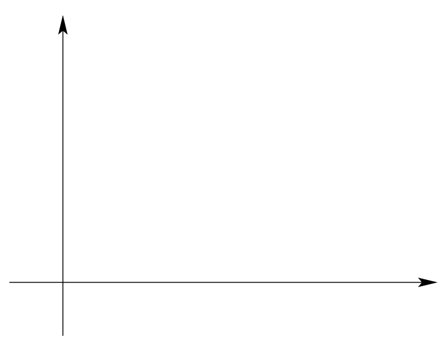
Section 4.1 Vectors in the Plane and in 3-Space

Definitions of scalar and vector

- Measurable quantities that can be completely described by giving their magnitude are called ____________ For example, ________________________.

- Measurable quantities that require for description not only magnitude, but also a sense of direction, are called ____________ For example, ________________________.

Vector in Plane



- A pair of perpendicular lines intersect at a point O, which is called the ____________.
- The horizontal line is called ____________, and the vertical line is called ____________
- The x- and y- axes together are called ____________, and they form a ________________________ or a ________________________.
- With each point P in the plane, we associate an order pair (x, y) of real numbers, its ____________, and denoted by ____________.
- Draw a direct line segment from O to P, denoted by ____________. Here O is called its ____________ and P is called its ____________.
- The line segment has a ____________, indicated by the arrow at its head. The length of the line segment is called the ____________.

Definition

A **vector in the plane** is

Remark Two vectors are equal if and only if

Example 1. *Find the values of a and b such that the following vectors are equal:*

$$\begin{bmatrix} a+b \\ 2 \end{bmatrix} \quad \textit{and} \quad \begin{bmatrix} 3 \\ a-b \end{bmatrix}$$

- A directed line segment $\overrightarrow{PQ}$ from the point $P(x, y)$ to the point $Q(x', y')$ is also a ____________.
- The **head** and **tail** of this vector is ____________ and ____________, respectively. The vector $\overline{PQ}$ can be represented by

Remark Different direct lines

Vector Operations

Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be two vectors. Let c be a scaler (a real number).

- The **sum** of the vector $\mathbf{u}$ and $\mathbf{v}$ is

- The **scalar multiple** $c\mathbf{u}$ is

The vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is called ________________ and denoted by _____.

Parallelogram Law

Example 2. *Let*

$$\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \textit{and} \quad \mathbf{v} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Find $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, $2\mathbf{u}$, *and* $-\mathbf{u}$

Vector in Space

In space, there are three coordinate axes which are called x-, y-, and z- axes. There are two types of coordinate systems.

Right-Handed Coordinate System **Left-Handed Coordinate System**

Properties of vector in $\mathbb{R}^2$ and $\mathbb{R}^3$

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$, and c, d be real numbers.

1.

2.

3.

4.

5.

6.

7.

8.