MA 265 Lecture 12

Section 4.1  Vectors in the Plane and in 3-Space

Definitions of scalar and vector

- Measurable quantities that can be completely described by giving their magnitude are called __________ For example, ____________________.

- Measurable quantities that require for description not only magnitude, but also a sense of direction, are called __________ For example, ____________________.

Vector in Plane

- A pair of perpendicular lines intersect at a point $O$, which is called the __________.

- The horizontal line is called __________, and the vertical line is called __________

- The $x$- and $y$- axes together are called __________, and they form a __________________ or a __________.

- With each point $P$ in the plane, we associate an order pair $(x, y)$ of real numbers, its __________, and denoted by __________.

- Draw a direct line segment from $O$ to $P$, denoted by __________. Here $O$ is called its __________ and $P$ is called its __________.

- The line segment has a __________, indicated by the arrow at its head. The length of the line segment is called the __________.
Definition

A vector in the plane is

Remark  Two vectors are equal if and only if

Example 1. Find the values of $a$ and $b$ such that the following vectors are equal:

\[
\begin{bmatrix} a + b \\ 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 3 \\ a - b \end{bmatrix}
\]

• A directed line segment $\overrightarrow{PQ}$ from the point $P(x, y)$ to the point $Q(x', y')$ is also a __________.

• The head and tail of this vector is __________ and __________, respectively. The vector $\overrightarrow{PQ}$ can be represented by

Remark  Different direct lines
Vector Operations

Let \( \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \) and \( \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \) be two vectors. Let \( c \) be a scaler (a real number).

- The **sum** of the vector \( \mathbf{u} \) and \( \mathbf{v} \) is

- The **scalar multiple** \( c \mathbf{u} \) is

The vector \( \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) is called ___________ and denoted by __________.

Parallelogram Law

**Example 2.** Let

\[
\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}
\]

Find \( \mathbf{u} + \mathbf{v}, \mathbf{u} - \mathbf{v}, 2\mathbf{u}, \text{ and } -\mathbf{u} \)
Vector in Space

In space, there are three coordinate axes which are called $x$-, $y$-, and $z$- axes. There are two types of coordinate systems.

**Right-Handed Coordinate System** \hspace{1cm} **Left-Handed Coordinate System**

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Properties of vector in $\mathbb{R}^2$ and $\mathbb{R}^3$

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$, and $c$, $d$ be real numbers.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8.