MA 265 Lecture 13

Section 4.2 Vector Spaces

Definition of Vector Space

A __________________________ is a set V of elements on which we have two operations _______ and _______ defined with following properties:

(a) If $\mathbf{u}$ and $\mathbf{v}$ are any elements in V, then $\mathbf{u} \oplus \mathbf{v}$ is in V.
(V is closed under the operation $\oplus$)

1.

2.

3.

4.

(b) If $\mathbf{u}$ is any element in V, and c is a real number, then $c \odot \mathbf{u}$ is in V.
(V is closed under the operation $\odot$)

5.

6.

7.

8.

- The elements of V are called _____________, and elements of the set of real numbers $\mathbb{R}$ are called _____________.
- The operation $\oplus$ is called ___________________, and $\odot$ is called ___________________.
- The vector $\mathbf{0}$ (in item (3)) is called ___________________.
- The vector $-\mathbf{u}$ (in item (4)) is called ___________________.

Example 1. *The set* M_{mn} *of all* $m \times n$ *matrices with matrix addition as* $\oplus$ *and scalar multiplications as* $\odot$ *is a vector space.*

Example 2. *Let* V *be the set of all* 2×2 *matrices with trace (sum of diagonal elements) equal to zero; that is*

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is in } V \qquad \text{provided } Tr(A) = a + d = 0.$$

The operation $\oplus$ *is matrix addition and the operation* $\odot$ *is scalar multiplications. Is* V *a vector space?*

Example 3. *Let V be the set of all 2×2 with the $(1,1)$-th entry equal to 1; that is*

$$A = \begin{bmatrix} 1 & b \\ c & d \end{bmatrix} \text{ is in } V.$$

The operation $\oplus$ is matrix addition and the operation $\odot$ is scalar multiplications. Is V a vector space?

Example 4. *Let V be the set of all real numbers with the operations*

$$\mathbf{u} \oplus \mathbf{v} = \mathbf{u} - \mathbf{v}, \qquad c \odot \mathbf{u} = c\mathbf{u}$$

Is V a vector space?

Example 5. *Let V be the set of all integers; define $\oplus$ as ordinary addition and $\odot$ as ordinary multiplication. Is V a vector space?*