Section 4.3  Subspaces

Definition of Subspaces

Let $V$ be a vector space and $W$ a nonempty subset of $V$. If

To check $W$ is a subspace of $V$, we only need to check closure property for $\oplus$ and $\odot$ on $W$:

**Theorem** Let $V$ be a vector space with operation $\oplus$ and $\odot$, and let $W$ be a nonempty subset of $V$. Then $W$ a nonempty subset of $V$ if and only if

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**Example 1.** Every vector space has at least two subspaces:

**Example 2.** Let $P$ be the set of all polynomials.

- Let $P_2$ be the set of all polynomials of degree $\leq 2$. Is $P_2$ a subset of $P$?
- Let $Q_2$ be the set all polynomials of degree $= 2$. If $Q_2$ a subset of $P$?
Example 3. Which of the following subsets of $\mathbb{R}^2$ with usual operations of vector addition and scalar multiplication are subspaces?

1. $W_1$ is the set of all vectors of the form \[
\begin{bmatrix}
 x \\
y
\end{bmatrix}, \text{ where } x \geq 0.
\]

2. $W_2$ is the set of all vectors of the form \[
\begin{bmatrix}
 x \\
y
\end{bmatrix}, \text{ where } x \geq 0, \text{ and } y \geq 0.
\]

3. $W_3$ is the set of all vectors of the form \[
\begin{bmatrix}
 x \\
y
\end{bmatrix}, \text{ where } x = 0.
\]
Example 4. Let $W$ be the set of vectors in $\mathbb{R}^3$ of the form
\[
\begin{bmatrix}
a \\
b \\
a + b
\end{bmatrix},
\]
where $a$ and $b$ are any real numbers. Is $W$ a subspace of $\mathbb{R}^3$?

Example 5. Let $v_1$ and $v_2$ be fixed vectors in a space $V$. Let $W$ be the set of all vectors in the form $av_1 + bv_2$, where $a$ and $b$ are any real numbers. Is $W$ a subspace of $V$?