MA 265 Lecture 14

Section 4.3 Subspaces

Definition of Subspaces

Let V be a vector space and W a nonempty subset of V. If

To check W is a subspace of V, we only need to check **closure** property for $\oplus$ and $\odot$ on W:

Theorem Let V be a vector space with operation $\oplus$ and $\odot$, and let W be a nonempty subset of V. Then W a nonempty subset of V if and only if

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Example 1. *Every vector space has at least two subspaces:*

Example 2. *Let P be the set of all polynomials.*

- *Let P_2 be the set of all polynomials of degree ≤ 2. Is P_2 a subset of P?*
- *Let Q_2 be the set all polynomials of degree $= 2$. If Q_2 a subset of P?*

Example 3. *Which of the following subsets of* $\mathbb{R}^2$ *with usual operations of vector addition and scalar multiplication are subspaces?*

1. W_1 *is the set of all vectors of the form* $\begin{bmatrix} x \\ y \end{bmatrix}$, *where* $x \geq 0$.

2. W_2 *is the set of all vectors of the form* $\begin{bmatrix} x \\ y \end{bmatrix}$, *where* $x \geq 0$, *and* $y \geq 0$.

3. W_3 *is the set of all vectors of the form* $\begin{bmatrix} x \\ y \end{bmatrix}$, *where* $x = 0$.

Example 4. *Let W be the set of vectors in $\mathbb{R}^3$ of the form* $\begin{bmatrix} a \\ b \\ a+b \end{bmatrix}$*, where a and b are any real numbers. Is W a subspace of $\mathbb{R}^3$?*

Example 5. *Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be fixed vectors in a space V. Let W be the set of all vectors in the form $a\mathbf{v}_1 + b\mathbf{v}_2$, where a and b are any real numbers. Is W a subspace of V?*