MA 265 Lecture 15

Section 4.3 Subspaces (cont)

Definition of Linear Combination

Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\cdots$, $\mathbf{v}_k$ be vectors in a vector space V.

Example 1. *Every polynomial of degree* ≤ 2 *is a linear combination of* t^2, t, 1.

Example 2. *Show that the set of all vectors in* $\mathbb{R}^3$ *of the form* $\begin{bmatrix} a \\ b \\ a+b \end{bmatrix}$ *is a linear combination of* $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ *and* $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

Example 3. *In* $\mathbb{R}^3$, *let*

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Verify that the vector

$$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, *and* $\mathbf{v}_3$.

Example 4. *Consider the homogeneous system*

$$A\mathbf{x} = \mathbf{0}$$

where A is an $m \times n$ matrix. The set W of solutions is a subset of $\mathbb{R}^n$. Verify that W is a subspace of $\mathbb{R}^n$ (called ***solution space****).*

Remark The set of all solutions of the linear system $A\mathbf{x} = \mathbf{b}$, with $\mathbf{b} \neq \mathbf{0}$, is