MA 265 Lecture 18

Section 4.4 Span

Review of Linear Combination

If $\mathbf{v}_1$ and $\mathbf{v}_2$ are two vectors in a vector space V,

Definition of Span

If $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ is a set of vectors in a vector space V,

Example 1. *Let S be the set of 2×2 matrices given by*

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

Find span S

Theorem Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ be a set of vectors in a vector space V. Then

Example 2. *Let $S = \{t^2, t\}$ be a subset of the vector space $\mathcal{P}_2$ (polynomials of degree no more than 2).*

Definition Let S be a set of vectors in a vector space V. If every vector in V is a linear combination of the vectors in S,

Remark If span $S = V$, S is called

Example 3. *In $\mathbb{R}^3$, let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ Determine whether the $\mathbf{v} = \begin{bmatrix} 1 \\ 5 \\ -7 \end{bmatrix}$ vector belongs to span $\{\mathbf{v}_1, \mathbf{v}_2\}$.*

Example 4. *In $\mathcal{P}_2$, let*

$$\mathbf{v}_1 = 2t^2 + t + 2, \quad \mathbf{v}_2 = t^2 - 2t, \quad \mathbf{v}_3 = 5t^2 - 5t + 2, \quad \mathbf{v}_4 = -t^2 - 3t - 2$$

Determine whether the vector $\mathbf{v} = t^2 + t + 2$ *belongs to span* $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

Example 5. *Let V be the vector space $\mathbb{R}^3$. Let*

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Determine whether the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ spans V.

Example 6. *Consider the homogeneous linear system* $A\mathbf{x} = \mathbf{0}$ *where*

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \\ 4 & 4 & -1 & 9 \end{bmatrix}$$