MA 265 Lecture 19

Section 4.5 Linear Independence

Recall The set W of all vectors of the form $\begin{bmatrix} a \\ b \\ a+b \end{bmatrix}$ is a subspace of $\mathbb{R}^3$.

Example 1. *Show that each of the following sets is a spanning set for W*

$$S_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}, \qquad S_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \right\}$$

Definition of Linear Dependency

The vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ in a vector space V are said to be **linearly dependent** if

Remark

- $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ are linearly independent if,

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- If $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$,

Example 2. *Determine whether the vectors*

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix},$$

are linearly independent.

Example 3. *Are the vectors*

$$\mathbf{v}_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix},$$

in M_{22} are linearly independent?

Example 4. *Determine whether the vectors*

$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

are linearly independent.

Theorem Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ be a set of n vectors in $\mathbb{R}^n$.

Example 5. *Is* $S = \{[1 \ \ 2 \ \ 3], \ \ [0 \ \ 1 \ \ 2], \ \ [3 \ \ 0 \ -1]\}$ *a linearly independent set of vectors in* $\mathbb{R}^3$*?*

Theorem Let S_1, S_2 be finite subsets of a vector space. Let S_1 be a subset of S_2. Then

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