MA 265 Lecture 2

Section 1.2 Matrices

Definitions

- A rectangular array of $m \times n$ real or complex numbers arranged in m horizontal **rows** and n vertical **columns**:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \tag{1}$$

  is called an __________________. We say that the matrix A is ____________.

- The i**th row** and the j**th column** of A are

- If $m = n$, we say that A is a ________________________, and that the numbers a_{11}, a_{22}, $\cdots$, a_{nn} form the ____________ of A.

- The number a_{ij} is called the __________________ of A, or the __________________ of A. We can write the matrix A as

- An $n \times 1$ matrix is called an __________________ or a __________________ when n is understood.

- Two $m \times n$ matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are **equal** if

Example 1. *Examples of matrices*

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1+i & 4i \\ 2-3i & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

Example 2. *If*

$$\begin{bmatrix} a+2b & 2a-b \\ 2c+d & c-2d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -3 \end{bmatrix}$$

find $a, b, c,$ *and* d.

Matrix Operations

Addition: If $A = [a_{ij}]$ and $B = [b_{ij}]$ are both $m \times n$ matrices, then the **sum** $A + B$ is an $m \times n$ matrix $C = [c_{ij}]$ defined by

Example 3. *Let*

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 3 & -4 \end{bmatrix}.$$

Then

$$A + B =$$

Remark: the sum of A and B is defined only when ________________________________.

Scalar Multiplication: If $A = [a_{ij}]$ is an $m \times n$ matrix and r is a real number, then the **scalar multiple** of A by r is an $m \times n$ matrix $C = [c_{ij}]$ where

Example 4. *Let*

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 3 & -4 \end{bmatrix}.$$

Then

$$5A =$$

$$A - B =$$

More Definitions

- If $A_1, A_2, \cdots, A_k$ are matrices of the same dimension, and $c_1, c_2, \cdots, c_k$ are real numbers, then
$$c_1 A_1 + c_2 A_2 + \cdots + c_k A_k \tag{2}$$
is called a ______________________________ of $A_1, A_2, \cdots, A_k$.

  The numbers $c_1, c_2, \cdots, c_k$ are called __________________.

- The linear combination (2) can be expressed using summation notation:

- If $A = [a_{ij}]$ is an $m \times n$ matrix, then the __________________ of A is an $n \times m$ matrix $A^T = [a_{ij}^T]$ defined by
$$a_{ij}^T =$$

Example 5. *Let*

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \end{bmatrix}$$

Then

$$A^T =$$

Example 6. *Suppose A and B are both $m \times n$ matrices. Then*

$$(2A + 3B)^T =$$

$$(A^T)^T =$$

$$(A - 2B^T)^T =$$