# MA 265 Lecture 20

## Section 4.6 Basis and Dimension

### Definition of Basis

The vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ in a vector space $V$ are said to form a **basis** for $V$ if

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### Remark

**Example 1.** *In* $\mathbb{R}^3$, *the vectors* $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, *form a basis.*

### Remark

**Example 2.** *Show that the set* $S = \{t^2 + 1, t - 1, 2t + 2\}$ *is a basis for the vector space* $\mathcal{P}_2$*.*

**Example 3.** *Find a basis for the subspace* $V$ *of* $\mathcal{P}_2$*, consisting of all vectors of the form* $at^2 + bt + c$*, where* $c = a - b$*.*

**Theorem** If $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ is a basis for $V$,

**Proof**

**Theorem** If $S = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$, and $T = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_l\}$ are bases for $V$, then

**Example 4.** *Let*

$$S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}, \quad T = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

*Is $S$ a basis of $M_{22}$? Is $T$ a basis of $M_{22}$?*

**Example 5.** *Let* $V = \mathbb{R}^3$, $s = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$, *where* $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_5 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$. *Find a subset of* $S$ *that is a basis for* $\mathbb{R}^3$.