MA 265 Lecture 21

Section 4.6 Basis and Dimension (continued)

Definition of Dimension

The **dimension** of a nonzero vector space V is

Example 1. *What is the dimension of the following vector spaces*

- P_2
- M_{23}
- M_{mn}
- $V = span \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \right\}$
- *the subspace of M_{33} consisting of all diagonal matrices*

Maximal Independent Subset and Minimal Spanning Set

Let S be a set of vectors in V. A subset T of S is called a **maximal independent subset** of S if

If W is a set of vectors spanning V, then W is called a **minimal spanning set** for V if

Example 2. *Let* $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$, *where*

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

Find the maximal independent subsets of S

Example 3. *Let* V *be a vector space has dimension* n.

- *a maximal independent subset of* V *contains* ____________ *vectors.*
- *a minimal spanning set for* V *contains* ____________ *vectors.*
- *any subset of* $m > n$ *vectors must be* ________________________.
- *any subset of* $m < n$ *vectors cannot* ________________________.

Theorem If S is linearly independent set of vectors in V, then

Example 4. *Find a basis for* $\mathbb{R}^4$ *that contains the vectors*

$$\mathbf{v}_1 = [1 \;\; 0 \;\; 1 \;\; 0], \qquad \mathbf{v}_2 = [-1 \;\; 1 \;\; -1 \;\; 0]$$

Theorem Let V be an n-dimensional vector space.

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