MA 265 Lecture 23

Section 4.9 Rank of a Matrix

Definition Let

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

be an $m \times n$ matrix.

- The rows of A,

- The columns of A,

Remark If A and B are row equivalent matrices, then

We can use this remark to find a basis for a subspace spanned by a given set of vectors.

Example 1. *Find a basis for the subspace V of $\mathbb{R}_5$ that is spanned by $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ where*

$$\mathbf{v}_1 = [1 \ \ -2 \ 0 \ 3 \ \ -4], \quad \mathbf{v}_2 = [3 \ 2 \ 8 \ 1 \ 4], \quad \mathbf{v}_3 = [2 \ 3 \ 7 \ 2 \ 3], \quad \mathbf{v}_4 = [-1 \ 2 \ 0 \ 4 \ 3],$$

Remark

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Example 2. *Let V be the subspace of Example 1. Given that the vector $\mathbf{v} = [5 \ \ 4 \ \ 14 \ \ 6 \ \ 3]$ is in V, write $\mathbf{v}$ as a linear combinations of the base determined by Example 1.*

Definition The dimension of the row (column) space of A is called

Remark If A and B are row equivalent,

Example 3. *Compute the row rank of A given by*

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 & -4 \\ 3 & 2 & 8 & 1 & 4 \\ 2 & 3 & 7 & 2 & 3 \\ -1 & 2 & 0 & 4 & -3 \end{bmatrix}$$

Example 4. *Compute the column rank of A in Example 3.*

Theorem Let A be an $m \times n$ matrix. Then