# MA 265 Lecture 25

## Section 5.1 Length and Direction in $\mathbb{R}^2$ and $\mathbb{R}^3$

**Length of Vectors in $\mathbb{R}^2$**

Let $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be a vector in $\mathbb{R}^2$. The ____________ or ____________ of $\mathbf{v}$, denoted by $\|\mathbf{v}\|$, is

The distance between vectors $\mathbf{u}$ and $\mathbf{v}$ is defined as

**Length of Vectors in $\mathbb{R}^3$**

Let $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ be a vector in $\mathbb{R}^3$. The **length** of $\mathbf{v}$ is defined as

The distance between vectors $\mathbf{u}$ and $\mathbf{v}$ is defined as

**Direction**

We consider the **angle** $\theta$, $0 \leq \theta \leq \pi$ between two vectors.

In $\mathbb{R}^2$, we plot the angle of two vectors $\mathbf{u}$ and $\mathbf{v}$:

By **law of cosines**:

we have

Similarly, if $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, the angle between vectors $\mathbf{u}$ and $\mathbf{v}$ is

**Example 1.** *Let* $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ *and* $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. *Find the angle* $\theta$ *between these vectors.*

The **inner product**, or **dot product** of vectors $\mathbf{u}$ and $\mathbf{v}$ on $\mathbb{R}^2$ (or $\mathbb{R}^3$) are defined by

**Remark**

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**Definition** Two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$ or $\mathbb{R}^3$ are called

**Example 2.** *The vectors* $\mathbf{u} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ *and* $\mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ *are orthogonal.*

Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$, and $c$ be a scalar. The inner product satisfies:

1. 
2. 
3. 
4. 

**Unit Vector**

A vector in $\mathbb{R}^2$ or $\mathbb{R}^3$ whose length is 1 is called

If $\mathbf{v}$ is any nonzero vector, then a unit vector in the direction of $\mathbf{v}$ is

**Example 3.** *Let* $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$. *Find a unit vector in* $\mathbb{R}^3$ *which*

1. *is in the same direction as* $\mathbf{v}$
2. *is in the opposite direction as* $\mathbf{v}$
3. *is orthogonal to* $\mathbf{v}$
4. *has an angle of* $60^o$ *between* $\mathbf{v}$.