MA 265 Lecture 27

Section 5.4 Gram-Schmidt Process

In this section, we introduce a method to obtain an orthonormal basis for a finite dimensional inner product space, which is called **Gram-Schmidt Process**.

Question: Why do we want an orthonormal basis?

Example 1. *Let* $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ *be a basis for* $\mathbb{R}^3$*, where*

$$\mathbf{u_1} = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}, \mathbf{u_2} = \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \mathbf{u_3} = \begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}.$$

Note that S *is orthonormal. Write the vector* $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ *as a linear combination of vectors in* S.

Gram-Schmidt Process

Let V be an inner product space, and $W \neq \{\mathbf{0}\}$ an m-dimensional subspace of V. We can construct an orthonomal basis $T = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_m\}$ for W.

Note: We first find an orthogonal basis $T^* = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m\}$ for W.

Example 2. *Let W be the subspace of $\mathbb{R}^4$ with basis $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ where*

$$\mathbf{u_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{u_2} = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \mathbf{u_3} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

Transform S to an orthonormal basis $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$

Example 3. *Find an orthonormal basis for the subspace of* R_4 *consisting of all vectors* $[a \;\; b \;\; c \;\; d]$ *such that*

$$a - b - 2c + d = 0$$