MA 265 Lecture 28

Section 5.5 Orthogonal Complements

Definition. Let W be a subspace of an inner product space V.

- A vector $\mathbf{u}$ in V is said to be **orthogonal to** W if

- The set of all vectors in V that are orthogonal to all vectors in W is called

  __________________________ of W in V, and is denoted by _____________.

Example 1. *Let W be the subspace spanned by the vector*

$$\mathbf{w} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}.$$

- *Find the orthogonal complement $W^{\perp}$ of W.*
- *Find a basis for $W^{\perp}$.*

Let W be a subspace of an inner product space V.

- $W^{\perp}$
- $W \cap W^{\perp} =$
- $V =$

Example 2. *Let W be the subspace of $\mathbb{R}_5$ spanned by $\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_5$ where*

$$\mathbf{w_1} = [2 \quad -1 \quad 0 \quad 1 \quad 2], \quad \mathbf{w_2} = [1 \quad 3 \quad 1 \quad -2 \quad -4], \quad \mathbf{w_3} = [3 \quad 2 \quad 1 \quad -1 \quad -2],$$

$$\mathbf{w_4} = [7 \quad 7 \quad 3 \quad -4 \quad -8], \quad \mathbf{w_5} = [1 \quad -4 \quad -1 \quad -1 \quad -2].$$

Find a basis for $W^{\perp}$.

Projections

Let W be a subspace of an inner product space V with orthonormal basis $\{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_m\}$. For any vector $\mathbf{v}$ in V, there exist

Moreover, the vector $\mathbf{w}$ can be written as

which is called the

Remark If $\{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_m\}$ is an orthogonal basis for W, then

An illustration of orthogonal projection

The **distance** from $\mathbf{v}$ to the subspace W is given by

Example 3. *Let W be the subspace of $\mathbb{R}^3$ with orthonormal basis $\{\mathbf{w}_1, \mathbf{w}_2\}$ where*

$$\mathbf{w_1} = \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}, \quad \mathbf{w_2} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}.$$

Find the orthogonal projection of $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$. *Find the distance from* $\mathbf{v}$ *to* W.