Definition. Let $W$ be a subspace of an inner product space $V$.

- A vector $u$ in $V$ is said to be orthogonal to $W$ if

- The set of all vectors in $V$ that are orthogonal to all vectors in $W$ is called the orthogonal complement of $W$ in $V$, and is denoted by $W^\perp$.

Example 1. Let $W$ be the subspace spanned by the vector

$$w = \begin{bmatrix} 2 \\ -3 \\ -4 \end{bmatrix}.$$ 

- Find the orthogonal complement $W^\perp$ of $W$.
- Find a basis for $W^\perp$. 


Let $W$ be a subspace of an inner product space $V$.

- $W^\perp$

- $W \cap W^\perp =$

- $V =$

Example 2. Let $W$ be the subspace of $\mathbb{R}_5$ spanned by $w_1, w_2, \ldots, w_5$ where

\[
\begin{align*}
  w_1 &= [2 \ -1 \ 0 \ 1 \ 2], \quad w_2 = [1 \ 3 \ 1 \ -2 \ -4], \quad w_3 = [3 \ 2 \ 1 \ -1 \ -2], \\
  w_4 &= [7 \ 7 \ 3 \ -4 \ -8], \quad w_5 = [1 \ -4 \ -1 \ -1 \ -2].
\end{align*}
\]

Find a basis for $W^\perp$. 

---

MA 265 Lecture 28 page 2 of 4
Projections

Let $W$ be a subspace of an inner product space $V$ with orthonormal basis 
$\{w_1, w_2, \cdots, w_m\}$. For any vector $v$ in $V$, there exist

Moreover, the vector $w$ can be written as

which is called the

**Remark** If $\{w_1, w_2, \cdots, w_m\}$ is an orthogonal basis for $W$, then

An illustration of orthogonal projection

The **distance** from $v$ to the subspace $W$ is given by
Example 3. Let $W$ be the subspace of $\mathbb{R}^3$ with orthonormal basis \{\(w_1, w_2\)\} where

\[
\begin{align*}
  w_1 &= \begin{bmatrix} 2 \\ 3 \\ -1/3 \end{bmatrix}, &
  w_2 &= \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}.
\end{align*}
\]

Find the orthogonal projection of \(v = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}\). Find the distance from \(v\) to \(W\).