

# MA 265 Lecture 29

## Section 5.6 Least Squares

**Recall** An  $m \times n$  linear system  $A\mathbf{x} = \mathbf{b}$  is consistent if and only if

**Question** What can we do if the system  $A\mathbf{x} = \mathbf{b}$  is inconsistent?

The least square solution to the linear system  $A\mathbf{x} = \mathbf{b}$  is the solution to the system

**Remark** If  $A$  is an  $m \times n$  matrix,

**Example 1.** Determine the least square solution to  $A\mathbf{x} = \mathbf{b}$ , where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}.$$

Least square problems often arise in constructing a mathematical model from discrete data.

**Example 2.** *The following data shows U.S. per capita health care expenditures*

<i>Year</i>	<i>Per Capita Expenditures (in \$)</i>
<i>1960</i>	<i>143</i>
<i>1970</i>	<i>348</i>
<i>1980</i>	<i>1,067</i>
<i>1990</i>	<i>2,738</i>
<i>1995</i>	<i>3,698</i>
<i>2000</i>	<i>4,560</i>

- *Determine the line of best fit to the given data.*
- *Predict the per capita expenditure for the year 2005, 2010, and 2015.*