MA 265 Lecture 3

Section 1.3 Matrix Multiplication

Definitions

- The **dot product** or **inner product**, of the n-vectors in $\mathbb{R}^n$

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

  is defined as

  Example 1.

$$\mathbf{a} = \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{a} \cdot \mathbf{b} =$$

- **Matrix Multiplication**
  If $A = [a_{ij}]$ is an ____________ matrix and $B = [b_{ij}]$ is a ____________ matrix, then the product of A and B, is the ____________ matrix $C = [c_{ij}]$, defined by

$$c_{ij} =$$

Remark The product of A and B is defined only when

Example 2. *Compute the product matrix AB, where*

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} \quad \textit{and} \quad B = \begin{bmatrix} -2 & 5 \\ 4 & -3 \\ 2 & 1 \end{bmatrix}$$

Question: Let $A = [a_{ij}]$ be an $m \times p$ matrix, and $B = [b_{ij}]$ be a $p \times n$ matrix. Is the statement $AB = BA$ true?

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Matrix-Vector Product Written in Terms of Columns

Let $A = [a_{ij}]$ be an $m \times n$ matrix and $\mathbf{c}$ be an n-vector

$$\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

Then

$$A\mathbf{c} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$$=$$

Example 3. *Let*

$$A = \begin{bmatrix} 2 & -1 & -3 \\ 4 & 2 & -2 \end{bmatrix} \quad \textit{and} \quad \mathbf{c} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}.$$

Then

$$A\mathbf{c} =$$

Linear Systems

Consider the linear system of m equations in n unknowns:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\qquad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned} \tag{1}$$

It can be written in matrix form $A\mathbf{x} = \mathbf{b}$, where

The matrix A is called __________________ of the linear system (1).

The matrix $[A \mid \mathbf{b}]$ is called the __________________ of the linear system (1) and has the form

If $b_1 = b_2 = \cdots = b_m = 0$ in (1), the linear system is called a ________________________.

Note that the matrix-vector product $A\mathbf{x}$ can be expressed as

$$A\mathbf{x} = x_1\mathbf{a}_1 + x_1\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n$$

so that the linear system (1) can be written as

Therefore, $A\mathbf{x} = \mathbf{b}$ is **consistent** if and only if