MA 265 Lecture 32

Section 6.1 Linear Transformation

Definition Let V and W be vector spaces. A function $L : V \to W$ is called a __________________________ of V to W if

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Example 1. *Let A be an $m \times n$ matrix. Define a function $L : \mathbb{R}^n \to \mathbb{R}^m$ by $L(\mathbf{u}) = A\mathbf{u}$. Show that L is a linear transformation.*

Remark: Every matrix transformation is a linear transformation. In particular,

Reflection (w.r.t. x-axis):

Projection (into xy-plane):

Dilation:

Contraction:

Rotation (counterclockwise through an angle ϕ):

Example 2. *Determine whether* $L : \mathbb{R}^3 \to \mathbb{R}^3$ *defined by*

$$L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 + 1 \\ 2u_2 \\ u_3 \end{bmatrix}$$

is a linear transformation.

Example 3. *Let* $L : \mathbb{R}_2 \to \mathbb{R}_2$ *be defined by*

$$L([u_1 \quad u_2]) = [u_1^2 \quad 2u_2]$$

Is L a linear transformation.

Example 4. *Let* $L : \mathbb{R}_4 \to \mathbb{R}_2$ *be a linear transformation and let* $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ *be a basis for* $\mathbb{R}_4$*, where*

$$\mathbf{v}_1 = [1 \quad 0 \quad 1 \quad 0], \quad \mathbf{v}_2 = [0 \quad 1 \quad -1 \quad 2], \quad \mathbf{v}_3 = [0 \quad 2 \quad 2 \quad 1], \quad \mathbf{v}_4 = [1 \quad 0 \quad 0 \quad 1].$$

Suppose that

$$L(\mathbf{v}_1) = [1 \quad 2], \quad L(\mathbf{v}_2) = [0 \quad 3], \quad L(\mathbf{v}_3) = [0 \quad 0], \quad L(\mathbf{v}_4) = [2 \quad 0].$$

Let $\mathbf{v} = [3 \quad -5 \quad -5 \quad 0]$*. Find* $L(\mathbf{v})$*.*

Every linear transformation $L : \mathbb{R}^n \to \mathbb{R}^m$ is associated with an $m \times n$ matrix A.

Question: How to find the matrix A?

Let $\{\mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_n\}$ be natural basis for $\mathbb{R}^n$.

Example 5. *Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by*

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ 3x_2 - 2x_3 \end{bmatrix}$$

Find the standard matrix representing L.