MA 265 Lecture 36

Appendix B.1 Complex Numbers

A **complex number** c is of the form ____________, where

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Example 1. *For symbol* $i = \sqrt{-1}$*, we have the following:*

$$i^2 = \qquad i^3 = \qquad i^4 = \qquad i^5 = \qquad i^6 = \qquad i^7 =$$

Operations

Let $c_1 = a_1 + b_1 i$ and $c_2 = a_2 + b_2 i$ be complex numbers.

- $c_1 + c_2 =$
- $c_1 - c_2 =$
- $c_1 c_2 =$

If $c = a + bi$ is a complex number, then the **conjugate** of c is

- $\bar{\bar{c}} =$
- $\overline{c_1 + c_2} =$
- $\overline{c_1 c_2} =$
- if $\bar{c} = c$, then
- $c\bar{c} =$

If $c_1 = a_1 + b_1 i$ and $c_2 = a_2 + b_2 i$, then

$$\frac{c_1}{c_2} =$$

Example 2. *Let* $c_1 = 2 - 5i,\ c_2 = -3 + 4i.$

$$c_1 + c_2 =$$

$$c_1 - c_2 =$$

$$c_1 c_2 =$$

$$\frac{c_1}{c_2} =$$

Example 3. *Find the value of following quantities*

$$\frac{1}{2+3i} =$$

$$\frac{1}{i} =$$

The **absolute value** or **modulus** of a complex number $c = a + bi$ is

Remark: $c\bar{c} =$

Matrices with Complex Entries

Example 4. *Let*

$$A = \begin{bmatrix} 4+i & -2+3i \\ 6+4i & -3i \end{bmatrix} \quad B = \begin{bmatrix} 2-i & 3-4i \\ 5+2i & -7+5i \end{bmatrix} \quad C = \begin{bmatrix} 1+2i & i \\ 3-4i & 8 \\ 4+2i & 1-i \end{bmatrix}$$

Find $A+B$, CA, *and* $(2+i)B$.

If $A = [a_{ij}]$, then the **conjugate** of A is

Properties of Conjugate of a Matrix

- $\overline{\overline{A}} =$
- $\overline{A+B} =$
- $\overline{AB} =$
- For any complex number c, $\overline{cA} =$
- $(\overline{A})^T =$

An $n \times n$ complex matrix A is called **Hermitian** if

For example, $A = \begin{bmatrix} 2 & 3+i \\ 3-i & 5 \end{bmatrix}$