MA 265 Lecture 38

Section 7.3 Diagonalization of Symmetric Matrices

In this section, we consider diagonalization of symmetric matrices since they are easier to handle and they arise in many applications.

Two Properties of Symmetric Matrices

- All eigenvalues of a symmetric matrix are

- Eigenvectors belonging to distinct eigenvalues are

Example 1. *Find eigenvalues and eigenvectors of the matrix*

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & -3 \end{bmatrix}$$

Orthogonal Matrix

Eigenvectors of a symmetric matrix is orthogonal; hence,

Definition A real square matrix A is called __________________, if

Example 2. *Let A be the matrix defined in Example 1. Find an orthogonal matrix P such that $D = P^{-1}AP$.*

Several Results Involving Orthogonal Matrices

1. A is orthogonal if and only if

2. If A is an orthogonal matrix, $det(A) =$

3. If A is a symmetric matrix,

Example 3. *Find an orthogonal matrix P such that such that $D = P^{-1}AP$ where*

$$A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$