# MA 265 Lecture 39

## Section 8.4   Differential Equations

In this section, we consider the first order differential equations.

A simple example of a differential equation is

$$\frac{d}{dt}x(t) = 2x(t).$$

The first order homogeneous system of differential equations is

$$
\begin{aligned}
x_1'(t) &= a_{11}x_1(t) + a_{12}x_2(t) + \cdots a_{1n}x_n(t) \\
x_2'(t) &= a_{21}x_1(t) + a_{22}x_2(t) + \cdots a_{2n}x_n(t) \\
&\vdots \\
x_2'(t) &= a_{n1}x_1(t) + a_{n2}x_2(t) + \cdots a_{nn}x_n(t)
\end{aligned}
$$

**Example 1.** *Find the solution of the following system of differential equations*

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

In general $A$ is not a diagonal matrix. Suppose that $A$ is diagonalizable.

**Example 2.** *Find the solution of the following system of differential equations*

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & -14 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

**Example 3.** *Find the solution of the following system of differential equations*

$$
\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
$$