MA 265 Lecture 4

Section 1.4 Algebraic Properties of Matrix Operations

1. Properties of Matrix Addition

Let A, B, and C be $m \times n$ matrices

(a) $A + B =$

(b) $A + (B + C) =$

(c) There is a unique $m \times n$ **zero matrix**, denoted by O, such that

(d) For each $m \times n$ matrix A, there is a unique $m \times n$ matrix D such that

2. Properties of Matrix Multiplication

Let A, B, and C be matrices of appropriate sizes.

(a) $A(BC) =$

(b) $(A + B)C =$

(c) $C(A + B) =$

Remark:

Example 1. *Let*

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 3 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 3 & -1 \end{bmatrix}, \quad \textit{and} \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}$$

Compute $(A+B)C$ *and* $AC+BC$.

3. Properties of Scalar Multiplication

Let r and s be real numbers. Let A and B be matrices of appropriate sizes.

(a) $r(sA) =$

(b) $(r+s)A =$

(c) $r(A+B) =$

(d) $A(rB) =$

4. Properties of Transpose

Let r and s be real numbers. Let A and B be matrices of appropriate sizes.

(a) $(A^T)^T =$

(b) $(A+B)^T =$

(c) $(AB)^T =$

(d) $(rA)^T =$

Questions

1. Does $A^2 = O$ imply $A = O$?

2. Does $AB = AC$ imply $B = C$?

Section 1.5 Special Types of Matrices

Diagonal Matrices

- An $n \times n$ matrix $A = [a_{ij}]$ is called a __________________ if

  Question: Is the zero matrix O a diagonal matrix?

- If the diagonal elements of a diagonal matrix are equal, we call it a __________________.
- If the diagonal elements of a diagonal matrix and are equal to 1, we call it a

  __________________ and write it as ____________.

  Property: $AI_n =$ $I_mA =$

Symmetric Matrices

- An $n \times n$ matrix $A = [a_{ij}]$ is called __________________ if

  It is called ________________________ if

- A matrix A with real entries is called __________________ if __________________.
- A matrix A with real entries is called ________________________ if __________________.

  Property: Every square matrix can be decomposed as the sum of a symmetric matrix and a skew symmetric matrix.

  Proof:

Nonsingular Matrices

An $n \times n$ matrix A is called ____________________ or ____________________ if

Such a matrix B is called an ____________________ of A and denoted by ______________.

If A is not invertible, we call it ____________________ or ____________________.

Theorem The inverse of a matrix, if

Proof:

Example 2. *Let*

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Find the inverse A^{-1} and B^{-1}, if they exist.

Properties of Inverse

(a) $(A^{-1})^{-1} =$

(b) $(AB)^{-1} =$

(c) $(A^T)^{-1} =$

Application to Linear System

If A is an $n \times n$ matrix, then the linear system $A\mathbf{x} = \mathbf{b}$ is a system of n equation in n unknowns.